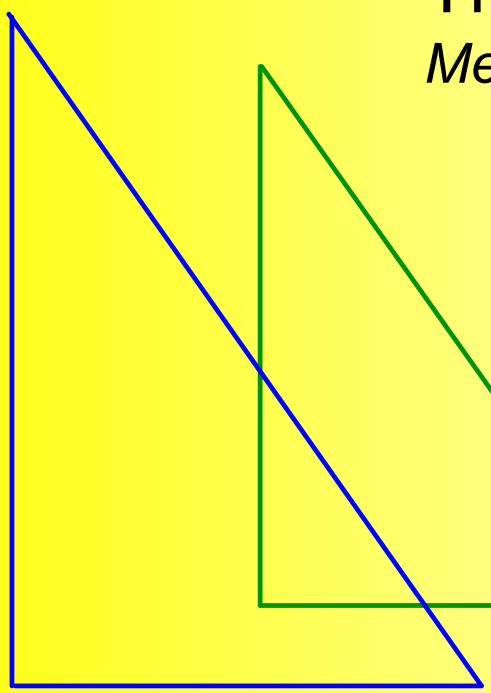
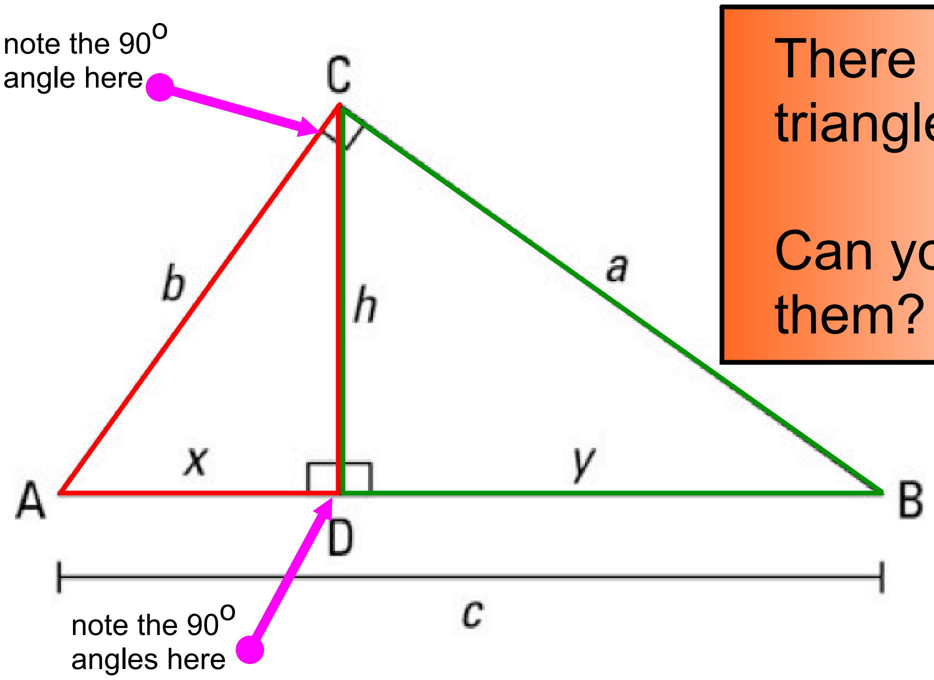
Triangle Geometry

Metric Relations

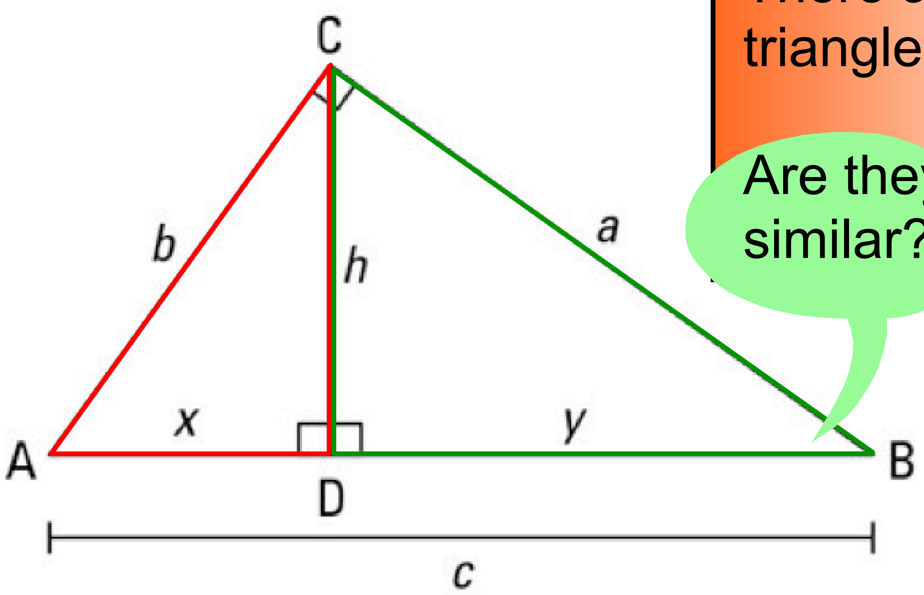




There are 3 triangles here.

Can you name them?

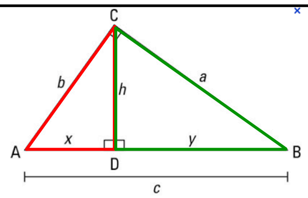
$\triangle ABC$ $\triangle ADC$ $\triangle CDB$



There are 3 triangles here.

Are they similar?

They **are** similar.



to prove that
ΔADC ΔCDB
show that the red is to the **large** and
the green is to the **large**, then
the red must be to the green.

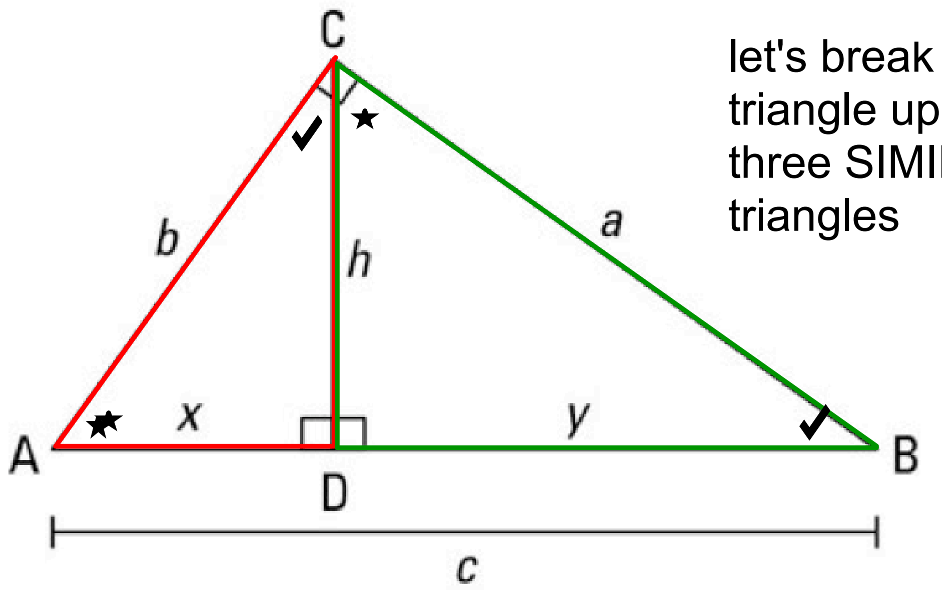
This is called the **transitive property**
if a=b and b=c then a=c

Proof:

$\measuredangle ADC \cong \measuredangle ACB$ (given)
$\measuredangle A \cong \measuredangle A$ (relexivity)
$\Delta ADC \sim \Delta ABC$ (AA)

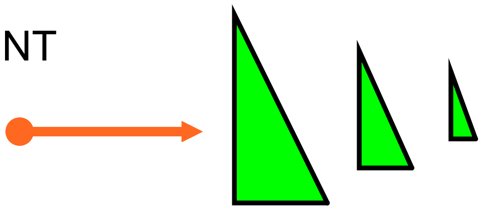
$\measuredangle CDB \cong \measuredangle ACB$ (given)
$\measuredangle B \cong \measuredangle B$ (relexivity)
$\Delta CDB \sim \Delta ABC$ (AA)

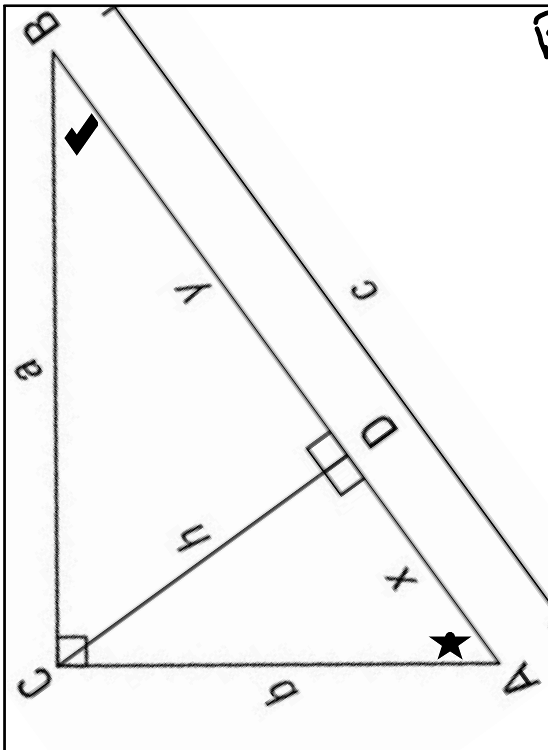
Since, $\Delta ADC \sim \Delta ABC$
and $\Delta ABC \sim \Delta CDB$
then
$\Delta ADC \sim \Delta CDB$ (transivity)

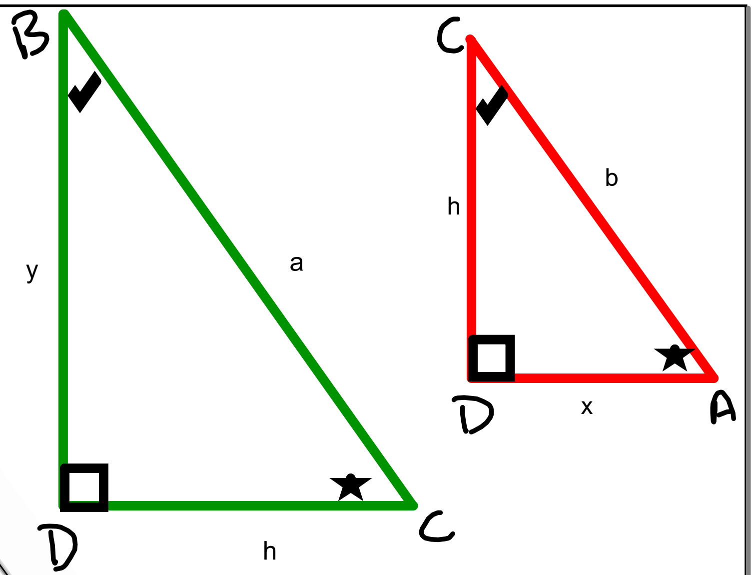


let's break this BIG triangle up into the three SIMILAR triangles

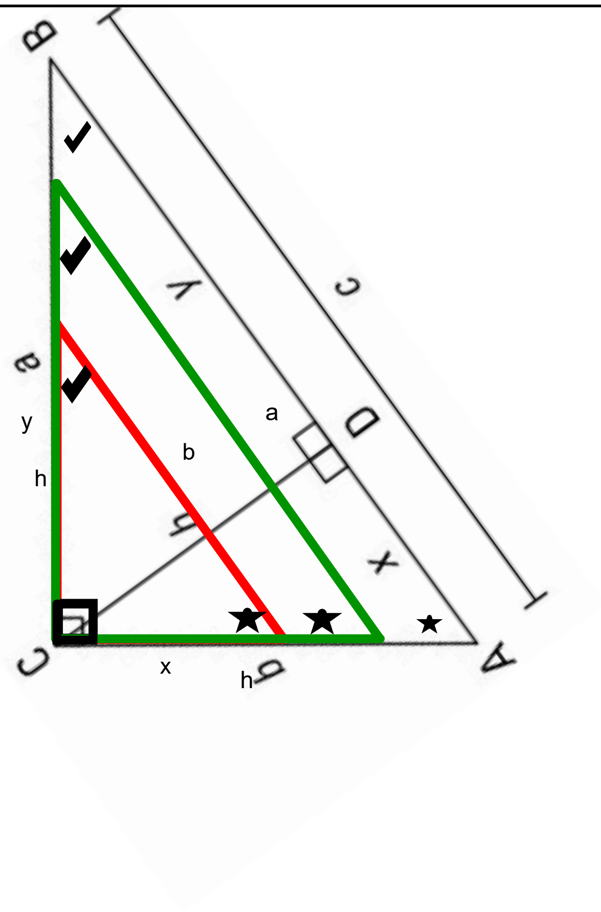
Be sure the ORIENT them in the same way...like this







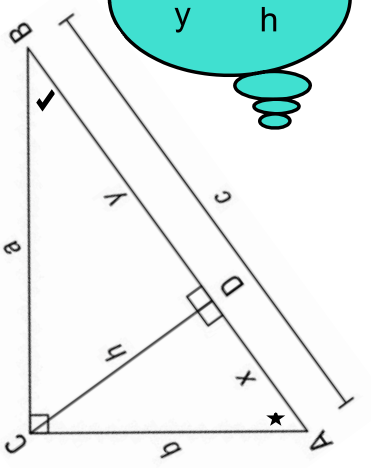
All 3 laying
on top
of each
other

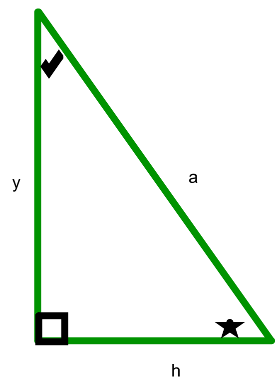


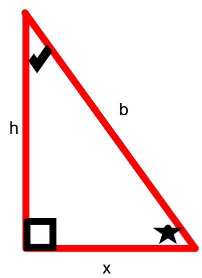
imagine all the proportions (ratios) you can create with **corresponding** sides!

ex.

$$\frac{a}{y} = \frac{b}{h}$$







write all the proportions between triangle **1** and 2	write all the proportions between triangle 2 and **3**	write all the proportions between triangle **1** and **3**

There are three proportions for each pair of triangles for a total of 9

black to green

$\frac{a}{y} = \frac{b}{h}$

$\frac{a}{y} = \frac{c}{a}$

$\frac{b}{h} = \frac{c}{a}$

black to red

$\frac{a}{h} = \frac{c}{b}$

$\frac{a}{h} = \frac{b}{x}$

$\frac{c}{b} = \frac{b}{x}$

green to red

$\frac{y}{h} = \frac{a}{b}$

$\frac{y}{h} = \frac{h}{x}$

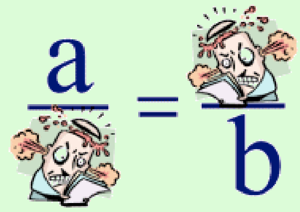
$\frac{h}{x} = \frac{a}{b}$

Are any of the 9 proportions "mean proportions"?

"**Mean Proportional**" may also be referred to as a "Geometric Mean".

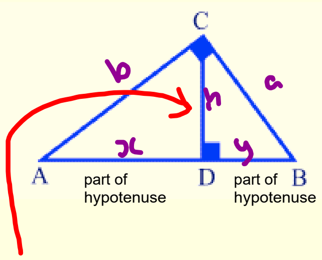
$$\frac{extreme}{mean} = \frac{mean}{extreme}$$

Remember the rule for working with proportions: the product of the means equals the product of the extremes.



In a mean proportional problem, the "means" are the same values.

The **altitude** to the hypotenuse of a right triangle is the mean proportional between the segments into which it divides the hypotenuse. 1.



$$\frac{AD}{CD} = \frac{CD}{DB}$$

altitude divides AB into 2 parts: AD and DB

Altitude Rule:

$$\frac{\text{part of hyp}}{\text{altitude}} = \frac{\text{altitude}}{\text{other part hyp}}$$

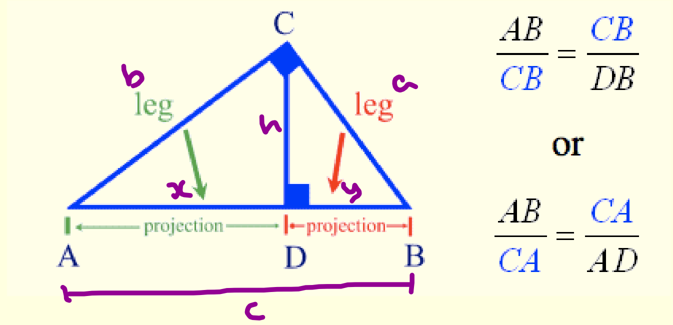
$$\frac{x}{h} = \frac{h}{y}$$

$$h^2 = xy$$

Mean Proportional (Altitude) Theorem

2.

Each **leg** of a right triangle is the mean proportional between the hypotenuse and the projection of the leg on the hypotenuse.



$$\frac{AB}{CB} = \frac{CB}{DB}$$

or

$$\frac{AB}{CA} = \frac{CA}{AD}$$

Leg Rule:

$$\frac{\text{hypotenuse}}{\text{leg}} = \frac{\text{leg}}{\text{projection}}$$

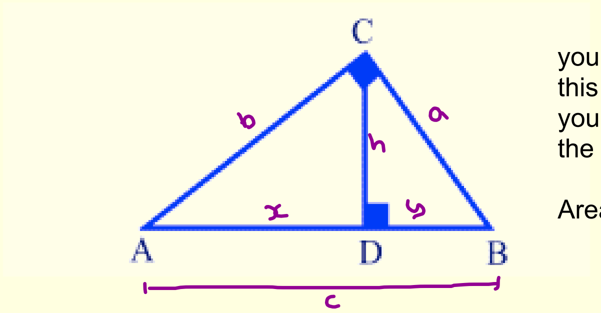
$\frac{c}{a} = \frac{a}{y}$ $\quad$ $\frac{c}{b} = \frac{b}{x}$

$a^2 = cy$ $\quad$ $b^2 = cx$

Mean Proportional (Leg) Theorem

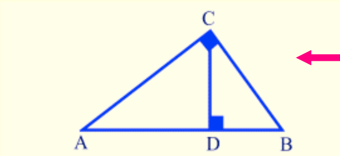
3.

The product of the sides of the right angle is equal to the product of the hypotenuse and the altitude (Product of the Sides Theorem)

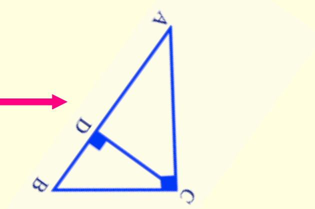


you actually already **know** this if you think about how you calculate the area of the triangle:

Area = ½ base x height



equal

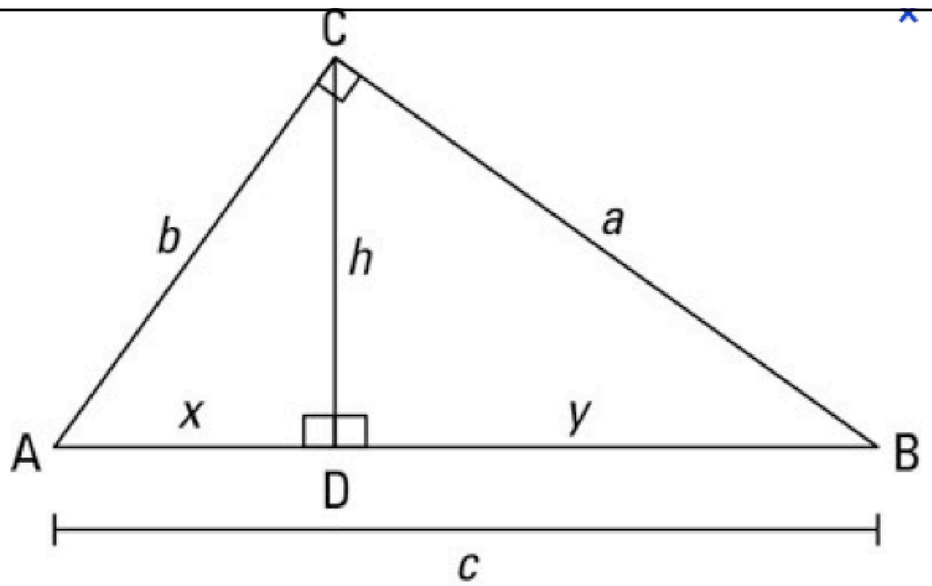


Area = ½ Hypotenuse x altitude $\qquad$ Area = ½ leg x leg

~~½~~ Hypotenuse x altitude = ~~½~~ leg x leg

Hypotenuse x altitude = leg x leg

$ch = ab$


C
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h
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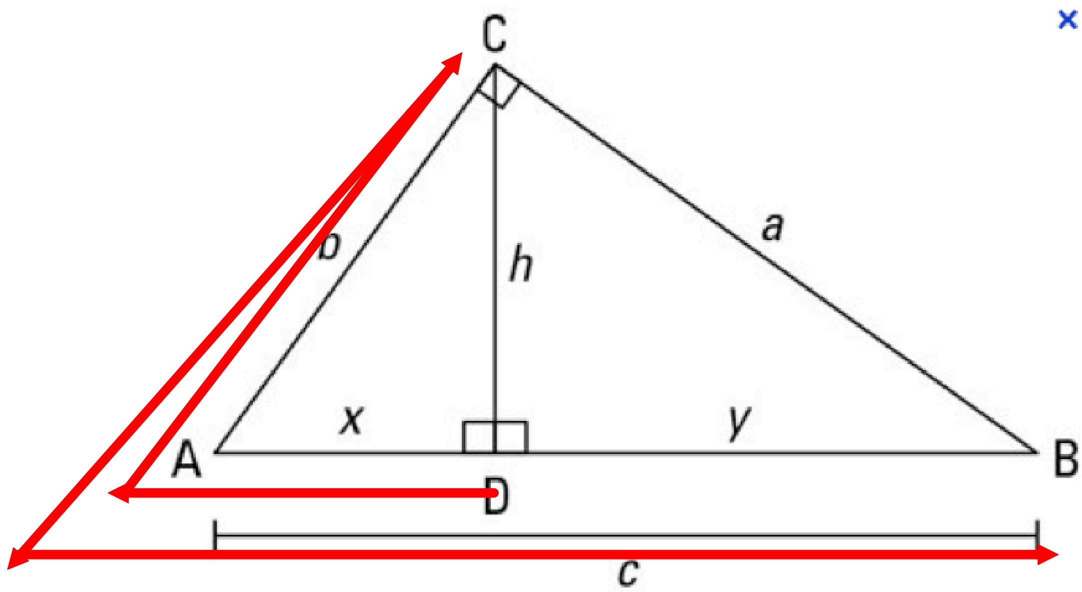
x
y
B
D
c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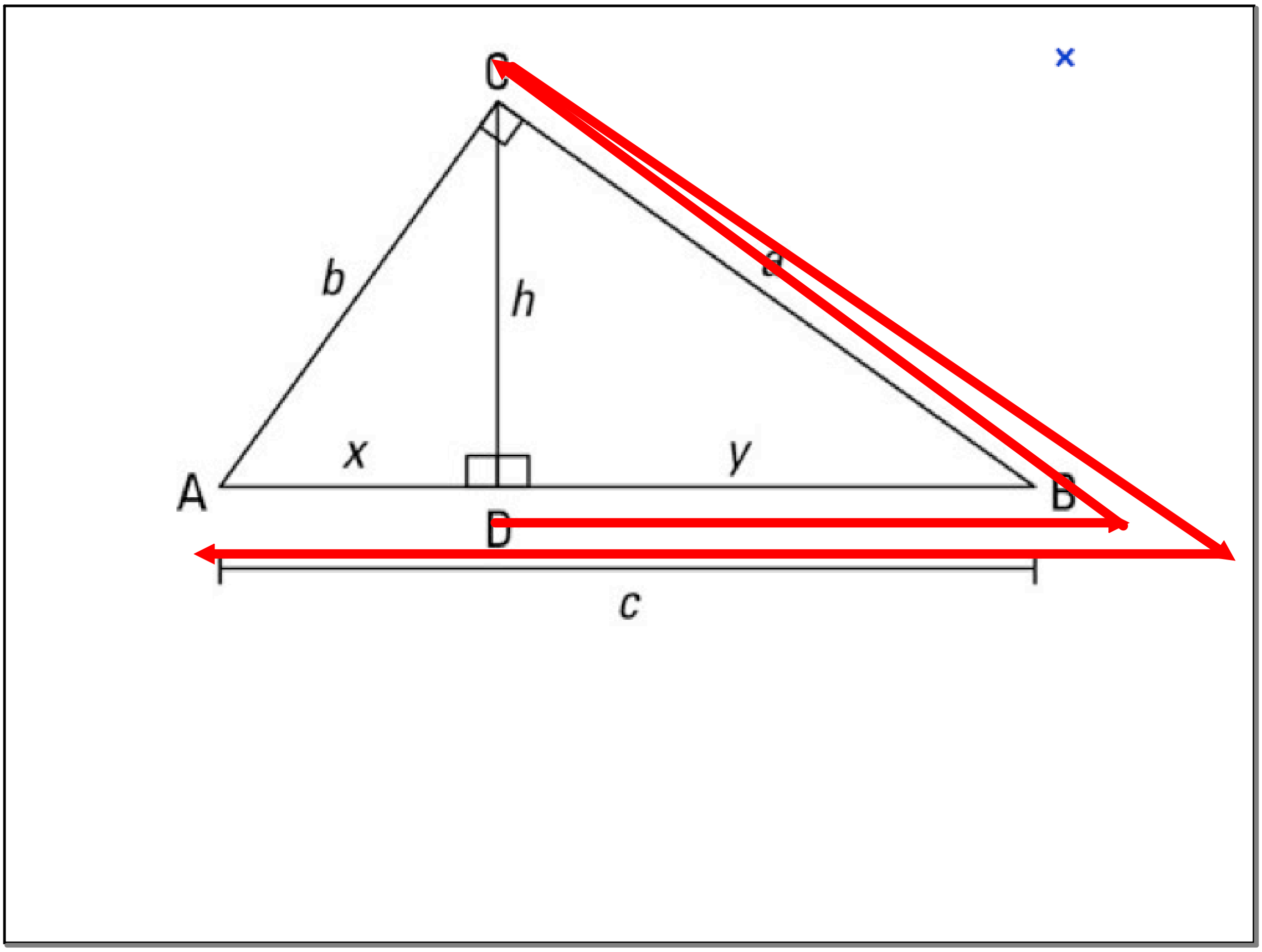
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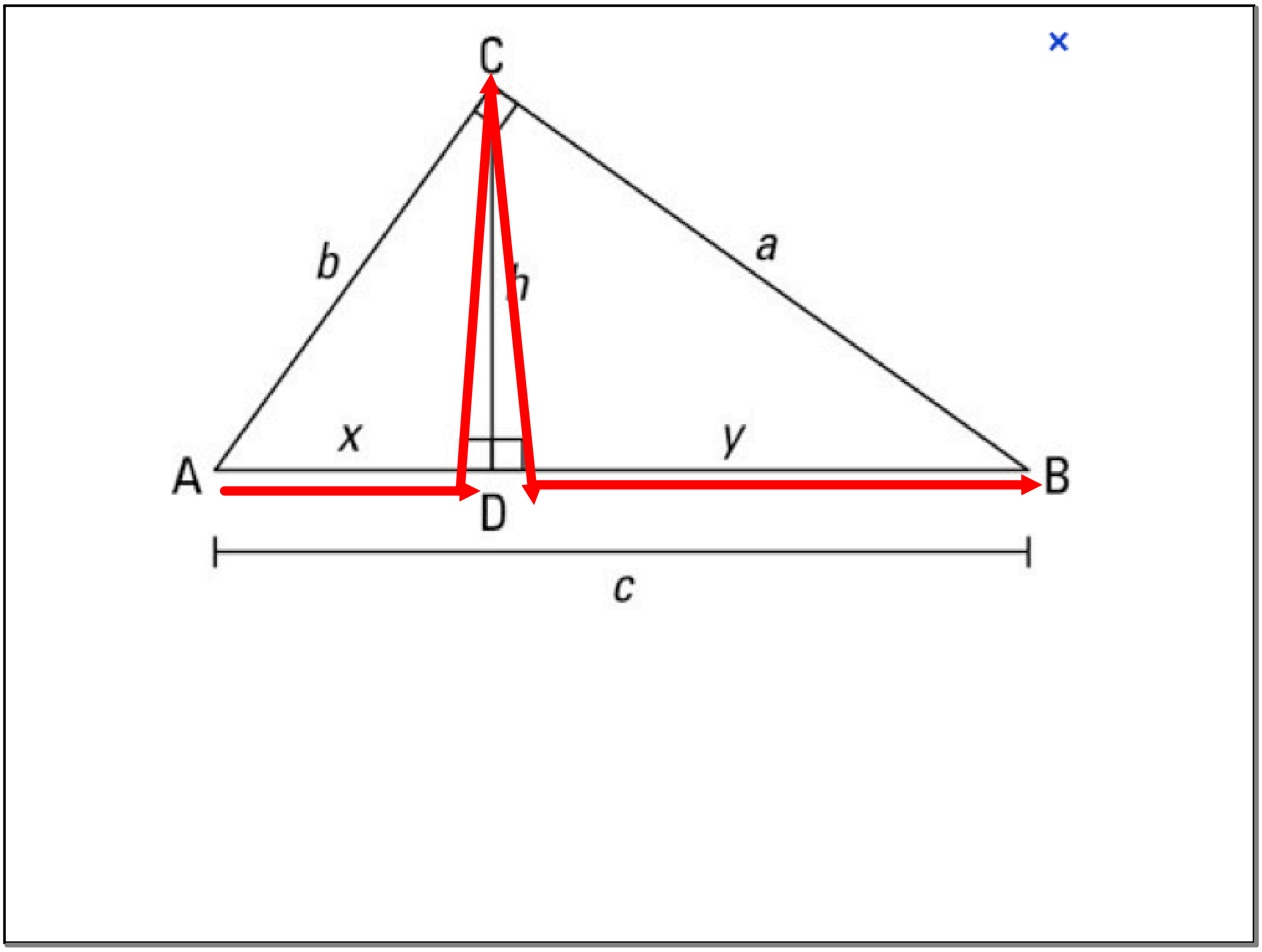
C
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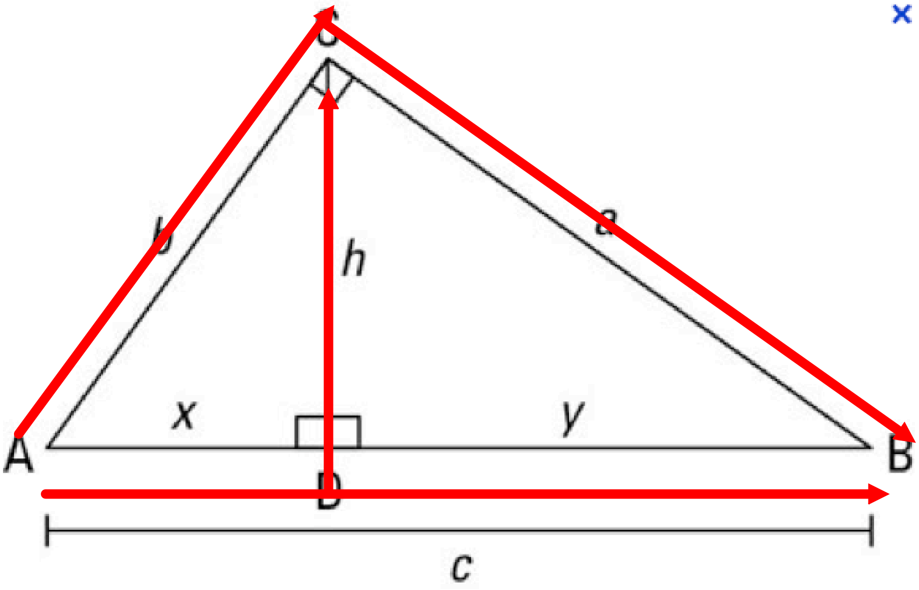
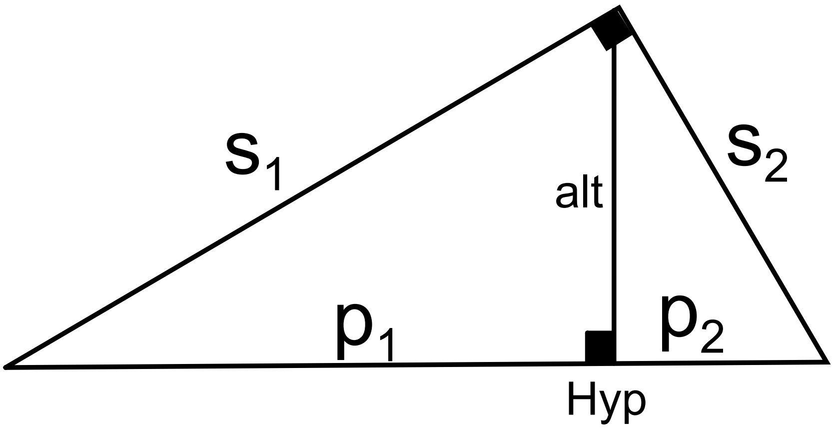




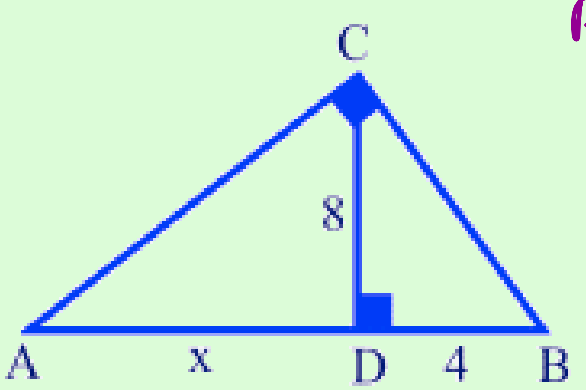
$S_1^2 = P_1 \, Hyp$

$S_2^2 = P_2 \, Hyp$

$Alt^2 = P_1 \bullet P_2$

$S_1 \cdot S_2 = Alt \cdot Hyp$

1. Find *x*:



Alt P_2 P_1

h x y

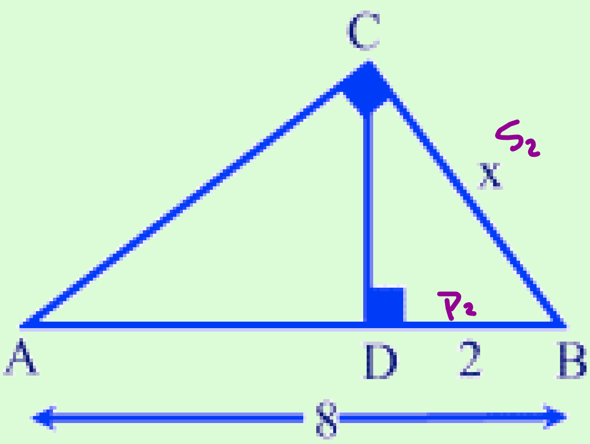
$Alt^2 = P_1 \bullet P_2$

$8^2 = 4x$

$\frac{64}{4} = \frac{4x}{4}$

$16 = x$

2. Find *x*:



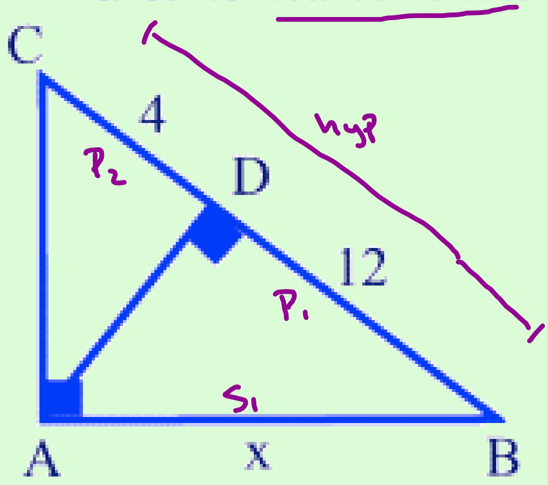
$S_2^2 = P_2 \cdot hyp$

$x^2 = 2(8)$

$\sqrt{x^2} = \sqrt{16}$

$x = 4$

3. Find *x* to *nearest tenth:*



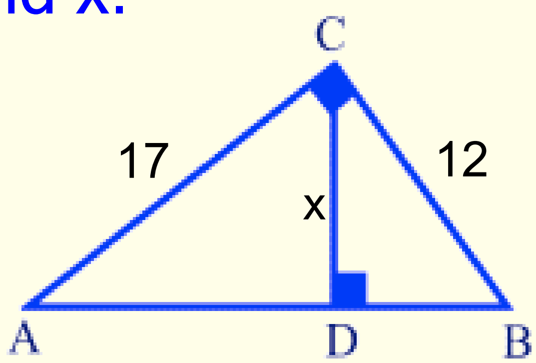
$s_1^2 = p_1 \cdot hyp$

$x^2 = 12(16)$

$\sqrt{x^2} = \sqrt{192}$

$x = 13.9$

4. Find x.



Homework:

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