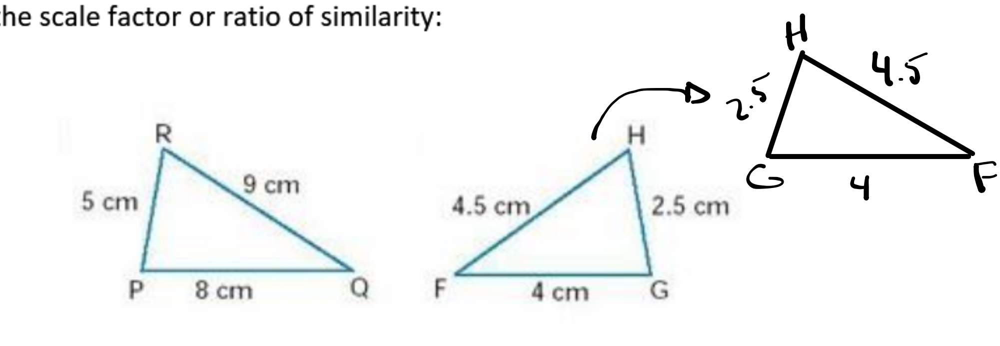
Similar Triangles

Find the scale factor or ratio of similarity:

$\frac{I}{O}$



Ratio of Similarity (k) = $\frac{Image}{Original}$

$K = \frac{2.5}{5} = \frac{\overline{GH}}{\overline{PR}}$ K = $\cancel{\frac{5}{2.5}}$

$K = \frac{4}{8} = \frac{\overline{GF}}{\overline{PQ}}$ K = $\cancel{\frac{8}{4}}$ $\Bigg\}\ \frac{1}{2}$

$K = \frac{4.5}{9} = \frac{\overline{HF}}{\overline{RQ}}$ K = $\cancel{\frac{9}{4.5}}$

Remember you need to compare the same sides in both triangles and check the ratios for each side. If they are all the same ratio, then the triangles are similar.

$\therefore$ = Therefore

$\because$ = Because

$$\frac{\overline{GH}}{\overline{PR}} = \frac{\overline{GF}}{\overline{PQ}}$$

$$\frac{2.5}{5} \;?\; \frac{4}{8}$$

$2.5(8) \stackrel{?}{=} 5(4)$

$20 = 20$ $\therefore$ Δ's are proportional $\therefore \Delta PQR \sim \Delta FGH$

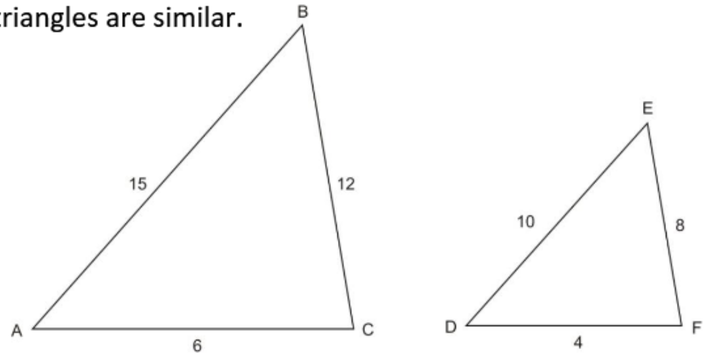
$\sim$ Similar

$\boxed{\cong \quad \text{Congruent}}$

There are three theorems that can justify whether or not two triangles are similar:

Theorem 1 – Side Side Side (SSS) Prop

If the scale factor or ratio between all three corresponding sides is the same, the triangles are similar.



$$K = \frac{DE}{AB} = \frac{10}{15} = \frac{2}{3}$$

$$K = \frac{EF}{BC} = \frac{8}{12} = \frac{2}{3}$$

$$K = \frac{DF}{AC} = \frac{4}{6} = \frac{2}{3}$$

$\frac{\overline{DE}}{\overline{AB}} = \frac{\overline{EF}}{\overline{BC}}$ $\frac{\overline{EF}}{\overline{BC}} = \frac{\overline{DF}}{\overline{AC}}$ $\frac{\overline{DF}}{\overline{AC}} = \frac{\overline{DE}}{\overline{AB}}$

$\frac{10}{15} \stackrel{?}{=} \frac{8}{12}$ $\frac{8}{12} \stackrel{?}{=} \frac{4}{6}$ $\frac{4}{6} \stackrel{?}{=} \frac{10}{15}$

$10(12) \stackrel{?}{=} 15(8)$ $8(6) \stackrel{?}{=} 12(4)$ $4(15) \stackrel{?}{=} 6(10)$

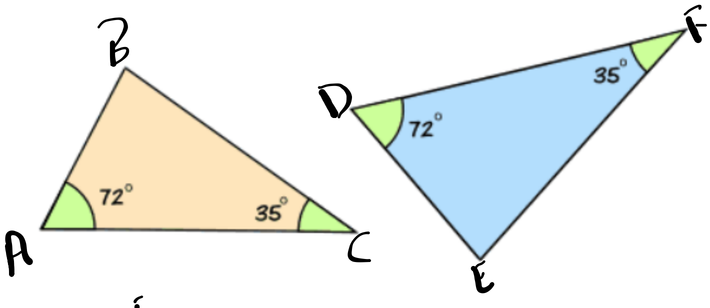
$120 = 120$ $48 = 48$ $60 = 60$

∴ all sides are proportional ∴ $\triangle ABC \sim \triangle DEF$

SSS

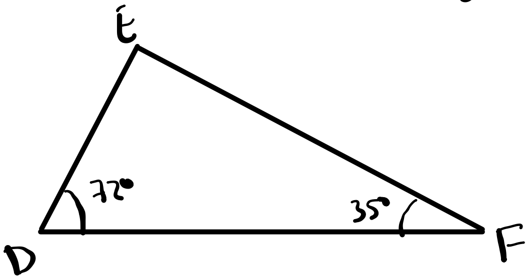
Theorem 2 – Angle Angle (AA)

If a pair of triangles have two angles in common, they are similar.



*Note – the sum of the interior angles in a triangle must add up to 180°

Therefore if two of the angles are the same, the 3rd is automatically the same



$\angle BAC \cong \angle EDF$

$\angle BCA \cong \angle EFD$

$\therefore \triangle ABC \sim \triangle DEF$

AA

Theorem 3 – Side Angle Side (SAS) prop

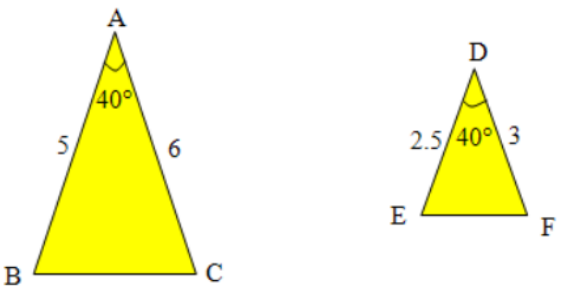
If one of the angles is the same in both triangles, and the ratio or scale factor between the two sides that form the angle (sandwich the angle) is the same, the two triangles are similar.

3 Checks:

k = $\frac{AB}{DE} = \frac{5}{2.5} = 2$

Angle A = Angle D

k = $\frac{AC}{DF} = \frac{6}{3} = 2$



$\frac{\overline{DE}}{\overline{AB}} \stackrel{?}{=} \frac{\overline{DF}}{\overline{AC}}$ $\frac{2.5}{5} \stackrel{?}{=} \frac{3}{6}$

$2.5(6) \stackrel{?}{=} 5(3)$

$15 = 15$

$\therefore \triangle ABC \sim \triangle DEF$ SAS_{prop}