Evaluation 3

1. Find the zeros of the following polynomial functions.

a) $f(x) = -2x + 7$ $\frac{7}{2}$

b) $f(x) = -2x^2 + 7x - 3$ $\frac{1}{2}$ and 3

c) $f(x) = -2(x - 3)^2 + 18$ 0 and 6

d) $f(x) = -4x^2 + 12x - 9$ $\frac{3}{2}$

2. Study the sign of the following functions.

a) $f(x) = -3x + 6$ $f(x) \geqslant 0$ if $x \in]-\infty, 2]$; $f(x) \leqslant 0$ if $x \in [2, +\infty[$

b) $f(x) = 2x^2 - 15x + 7$ $f(x) \geqslant 0$ if $x \in \left]-\infty, \frac{1}{2}\right] \cup [7, +\infty[$; $f(x) \leqslant 0$ if $x \in \left[\frac{1}{2}, 7\right]$

3. What is the range of the following functions?

a) $f(x) = 5(x - 1)^2 - 20$

ran $f = [-20, +\infty[$

b) $f(x) = -2x^2 + 18x - 21$

ran $f = \left]-\infty, \frac{39}{2}\right]$

4. Study the variation of the following functions.

a) $f(x) = -2x + 1$

$f \searrow, \forall\, x \in \mathbb{R}$

b) $f(x) = 3x - 1$

$f \nearrow, \forall\, x \in \mathbb{R}$

c) $f(x) = -2(x + 1)^2 + 9$

$f \nearrow$ if $x \in]-\infty, -1]$

$f \searrow$ if $x \in [-1, +\infty[$

d) $f(x) = x^2 - 6x + 8$

$f \searrow$ if $x \in]-\infty, 3]$

$f \nearrow$ if $x \in [3, +\infty[$

5. Find the rule of the following functions.

a) f is a constant function such that $f(1) = 2$. $f(x) = 2$

b) f is a linear function such that $f(1) = 2$ and $f(3) = 5$. $f(x) = \frac{3}{2}x + \frac{1}{2}$

c) f is a quadratic function with zeros -3 and 2 and an initial value of -12.

$f(x) = 2x^2 + 2x - 12$

d) f is represented by a parabola with the vertex V(2, 1) and passing through the point P(4, −7).

$f(x) = -2(x - 2)^2 + 1$

6. At the start of a car ride, the gas tank contains 66 litres. After traveling 50 km, the gas tank contains 60 litres. What is the rule of the linear function which gives the remaining quantity y of gas in the tank as a function of the distance traveled x in km?

$y = 66 - 0.12x$

7. A parabola with the vertex V(3, 16) passes through the point A(5, 12). What is its y-intercept? 7

8. A parabola intersects the x-axis at -2 and 4 and passes through the point A(2, −24). Find the coordinates of its vertex. V(1, −27)

9. The trajectory of a ball thrown by David is a partial parabola. The height $h(t)$, in metres, reached by the ball is described by the rule $h(t) = -(t-3)^2 + 9$.

Determine over what interval of time the ball is at a height greater than or equal to 8 m above ground.

[2, 4]

10. A share purchased for \$4 reaches its maximum value of \$4.50 five weeks after its purchase. The function associating the value $v(t)$, in dollars, of the share as a function of time t, in weeks, has been shown to be a quadratic function. What is the value of the share 8 weeks after its purchase?

$v(t) = -0.02(t-5)^2 + 4.50$; $v(8) = 4.32$

The share is worth \$4.32 eight weeks after its purchase.

11. The number of units $q(x)$ produced per day by x employees is given by the rule $q(x) = -0.25x^2 + 10x$ $(x \leq 25)$.

a) What is the maximum number of units produced in one day? How many employees are required to produce this maximum?

100 units produced by 20 employees.

b) How many employees are required to produce 75 units?

10 employees

12. A truck with a height of 190 cm enters a tunnel with a parabolic ceiling. The width of the tunnel is 20 m and the maximum height of the tunnel is 10 m. At what minimal distance from the edge at ground level can this truck pass through the tunnel? ***1 m***

13. The rule $p = 1000 - 2q$ enables you to calculate the selling price p of a parasol as a function of the number q of parasols ordered. What must the number of parasols ordered be to maximize the revenue generated by the sale of the parasols? ***250 parasols***

14. A stone is thrown vertically upward. The function which gives the height $h(t)$, in metres, as a function of elapsed time t, in seconds, since the stone was thrown is a quadratic function with the rule: $h(t) = -2t^2 + 12t$.

a) 1. What is the maximum height reached by the stone? ***18 m***

2. At what time does the stone reach its maximum height? ***3 seconds***

b) At what time, during its descent, does the stone reach a height of 16 m?

4 seconds after it was thrown.

15. A rectangular yard is to be fenced in with 80 m of fence. What must the dimensions of the yard be in order to maximize the area of the field?

x: width of the yard; 40 – x: length of the yard

A(x): area of the yard. $A(x) = -x^2 + 40x$

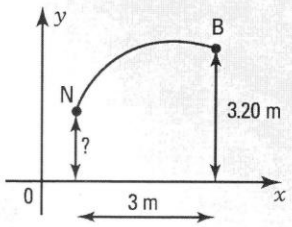
The vertex of the parabola representing A(x) has the coordinates V(20, 400).

The yard must be in the shape of a square with 20 m sides.

16. Nancy throws a ball toward a basket located 3.2 m off the ground. The ball's trajectory is represented on the right.

The rule associated with this trajectory is: $y = -0.4(x - 6)^2 + 3.6$.

Nancy throws the ball at a distance of 3 m from the basket. From what height did Nancy throw the ball?

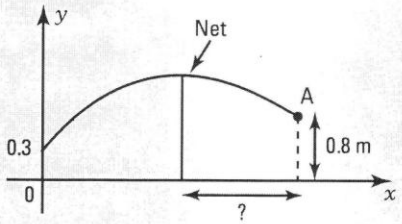


$y_B = 3.20$; $x_B = 7$; $x_N = 4$; $y_N = 2$

The ball is thrown from a height of 2 m.

17. During a tennis match, Karen hits the ball to Alex. The trajectory of the ball is represented in the Cartesian plane by a parabola with its vertex over the net. The equation of the trajectory is: $f(x) = -0.1(x - 3)^2 + 1.2$.

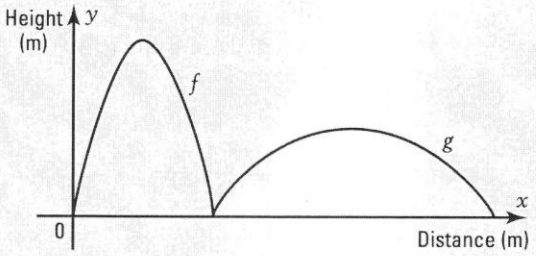
The ball is hit by Karen at a height of 0.3 m and reaches Alex at a height of 0.8 m on its descent. How far is Alex from the net if the vertex of the ball's trajectory is directly over the net?



$x_A = 5$; $x_V = 3$; $x_A - x_V = 2$. Alex is located 2 m from the net.

18. A kangaroo makes two consecutive jumps. The trajectory is represented by two portions of parabolas associated with the functions f and g.

The rule associated with the second jump is $g(x) = -0.25(x - 6.4)^2 + 2.56$. What is the rule associated with the first jump if the kangaroo jumped twice as high on the first jump as the second jump? (The variables x and y are expressed in metres.)



The zeros of g are 3.2 and 9.6.

Vertex of the 1st parabola: V(1.6, 5.12); $f(x) = -2(x - 1.6)^2 + 5.12$.