Lesson 20

Factoring Polynomials
Completing the Square

Completing the Square Method

If the trinomial is not a perfect square we can complete the square by adding a constant.

NB: This method only works when the coefficient of x^2 is 1. If it is not, the expression must be factored (or the equation must be divided by the coefficient) before we can complete the square.

$a^2 \pm 2ab + b^2$

Ex. 1 $2x^2 - 12x + 8 = 2(x^2 - 6x + 4)$

Completing the Square Method

To complete the square $x^2 + bx$, add the square of half the coefficient of x (ie. $\left(\frac{b}{2}\right)^2$) to create a perfect square. We must also subtract this same constant from the expression.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = (x + \frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c$$

Completing the Square Method

Ex. 1 Factor $2x^2 - 12x + 6$

$$= 2(x^2 - 6x + 3)$$

$$= 2[(x^2 - 6x + 9) - 9 + 3]$$

$$= 2[(x-3)^2 - 6]$$

$$f(x) = 2(x-3)^2 - 12$$

Step 1: Make the coefficient of x^2=1

Step 2: Find $\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = 9$

Step 3: Add (and subtract) this constant

Step 4: Factor the perfect square

Step 5: Simplify

Completing the Square Method

Ex. 2 Factor $2x^2+5x+4$

$a^2+2ab+b^2$
$(a+b)^2$

$$=2(x^2+\frac{5}{2}x+2)$$

$$=2[(x^2+\frac{5}{2}x+\frac{25}{16})-\frac{25}{16}+2]$$

$$=2[(x+\frac{5}{4})^2+\frac{7}{16}]$$

$$=2(x+\frac{5}{4})^2+\frac{7}{8}$$

$a(x-h)^2+k$

Step 1: Make the coefficient of x^2=1

Step 2: Find $\left(\frac{b}{2}\right)^2=\left(\frac{5/2}{2}\right)^2=\frac{25}{16}$

Step 3: Add (and subtract) this constant

Step 4: Factor the perfect square

Step 5: Simplify

Completing the Square Method

Ex. 3 Factor: $x^2-12x+2$

$(x^2-12x+36)-36+2$

$(x-6)^2-34$

$\left(\frac{b}{2}\right)^2=\left(\frac{-12}{2}\right)^2=36$

Completing the Square Method

This method is used mainly when solving equations.

Ex. 4 $2x^2 - 12x + 6 = 0$

$2(x^2 - 6x + 3) = 6$

$2(x-3)^2 - 12 = 0$ Step 1: Factor

$2(x-3)^2 = 12$ Step 2: Solve for x

$(x-3)^2 = 6$

$x - 3 = \pm\sqrt{6}$

Completing the Square Method

Ex. 4 $2x^2 - 12x + 6 = 0$

Case 1: $x - 3 = +\sqrt{6}$

$x = 3 + \sqrt{6}$

$x \approx 5.45$

Case 2: $x - 3 = -\sqrt{6}$

$x = 3 - \sqrt{6}$

$x \approx 0.55$

Completing the Square Method

Ex. 5 Solve: $2x^2 - 22x + 60 = 0$

$x = 5 \; or \; x = 6$

Completing the Square Method

Ex. 6 Solve: $18n^2 + 6n + 6 = 6n^2 + 30n - 3$

$x = \frac{1}{2} \; or \; x = \frac{3}{2}$