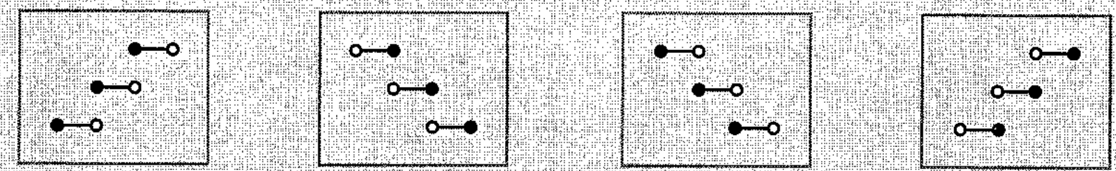
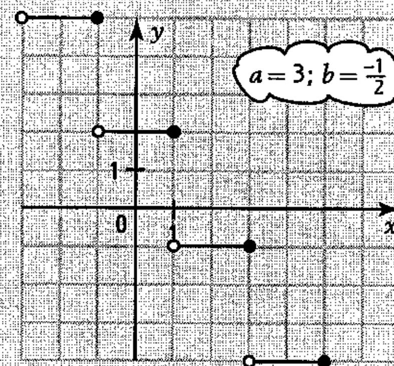
- The signs of a and b determine 4 cases:

$a > 0$ and $b > 0$ | $a > 0$ and $b < 0$ | $a < 0$ and $b > 0$ | $a < 0$ and $b < 0$



Ex.: $f(x) = 3\left[-\frac{1}{2}(x-1)\right] + 2$

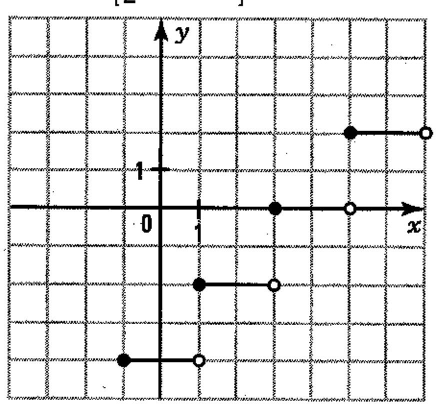


We have: $a = 3$; $b = \frac{-1}{2}$; $h = 1$ and $k = 2$.

- Each step has a length of $\frac{1}{|b|} = 2$.
- The counterstep height is $|a| = 3$.
- dom $f = \mathbb{R}$
- ran $f = \{y \mid y = 3m + 2, m \in \mathbb{Z}\}$
- zeros of f : f has no zeros since k is not a multiple of a.
- y-intercept of f: 2.
- $f(x) > 0$ if $x \leq 1$; $f(x) < 0$ if $x > 1$
- f is decreasing over $\mathbb{R}$, since $ab < 0$.
- f has no extrema.

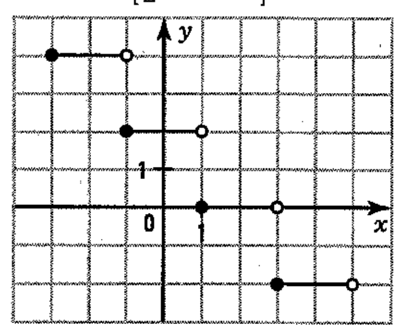
1. Represent the following functions and determine the set S of zeros.

a) $f(x) = 2\left[\frac{1}{2}(x-1)\right] - 2$



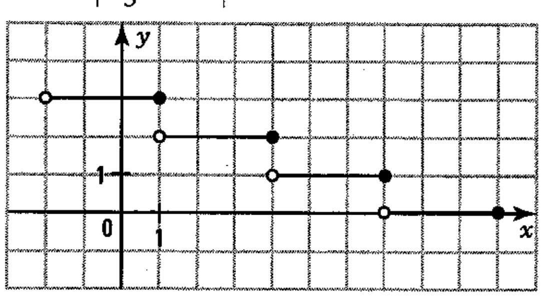
***S* = [3, 5[**

b) $f(x) = -2\left[\frac{1}{2}(x+1)\right] + 2$



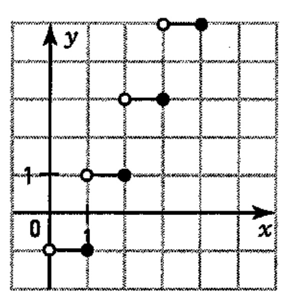
***S* = [1, 3[**

c) $f(x) = \left[-\frac{1}{3}(x-1)\right] + 3$



***S* =]7, 10]**

d) $f(x) = -2[-(x-2)] + 1$



***S* = ∅**

2. The following rules define greatest integer functions. Write them in the form $y = a[b(x - h)] + k$.

a) $y = [2x - 1]$ ***$y = \left[2\left(x - \frac{1}{2}\right)\right]$***

b) $y = -2[3x - 6]$ ***$y = -2[3(x - 2)]$***

c) $y = \left[\frac{x-3}{2}\right]$ ***$y = \left[\frac{1}{2}(x - 3)\right]$***

d) $y = \left[\frac{x}{3} - 1\right]$ ***$y = \left[\frac{1}{3}(x - 3)\right]$***

3. Which geometric transformations apply the basic greatest integer function to the following functions?

a) $f(x) = [x - 5]$ ***Horizontal translation of 5 units to the right.***

b) $f(x) = [x] + 2$ ***Vertical translation of 2 units upward.***

c) $f(x) = [x + 3] - 1$ ***Horizontal translation of 3 units to the left, followed by a vertical translation of one unit downward.***

d) $f(x) = -[x - 2]$ ***Horizontal translation of 2 units to the right, followed by a reflection about the x-axis.***

e) $f(x) = [-2x]$ ***Horizontal scale change followed by a reflection about the y-axis.***

4. For each of the following greatest integer functions, determine

1. the length of one step and its type (●—○) or (○—●).
2. the counterstep height.
3. the set S of zeros.
4. the y-intercept.
5. the variation of the function.

a) $y = 2[-3(x - 1)] + 4$

1. ***length: $\frac{1}{3}$; ○—●***
2. ***height: 2***
3. ***$S = \left]\frac{4}{3}, \frac{5}{3}\right]$***
4. ***y-int.: 10***
5. ***decreasing function***

b) $y = \left[\frac{1}{2}(x + 1)\right] + 6$

1. ***length: 2; ●—○***
2. ***height: 2***
3. ***$S = [-1, 1[$***
4. ***y-int.: 0***
5. ***increasing function***

c) $y = 3[2(x + 1)] - 5$

1. ***length: $\frac{1}{2}$; ●—○***
2. ***height: 3***
3. ***$S = \varnothing$***
4. ***y-int.: 1***
5. ***increasing function***

d) $y = \frac{1}{2}[-4(x + 1)] + 2$

1. ***length: $\frac{1}{4}$; ○—●***
2. ***height: $\frac{1}{2}$***
3. ***$S = \left]-\frac{1}{4}, 0\right]$***
4. ***y-int.: 0***
5. ***decreasing function***

5. Determine the domain and range of the following functions.

a) $y = -3[5(x + 2)] - 7$

dom = $\mathbb{R}$; ran = $\{y \mid y = -3m - 7, m \in \mathbb{Z}\}$

b) $y = \frac{1}{2}[-3(x - 1)] + 4$

dom = $\mathbb{R}$; ran = $\{y \mid y = \frac{1}{2}m + 4, m \in \mathbb{Z}\}$

6. Find the set of values of x for which

1. $f(x) \geqslant 0$.
2. $f(x) < 0$.

a) $f(x) = 3[3(x-1)] + 2$

1. $x \in [1, +\infty[$

2. $x \in [-\infty, 1[$

b) $f(x) = -3\left[\frac{1}{3}x + 2\right] + 6$

1. $x \in]-\infty, 3[$

2. $x \in [3, +\infty[$

c) $f(x) = -2\left[\frac{1}{4}(x-1)\right] - 4$

1. $x \in]-\infty, -3[$

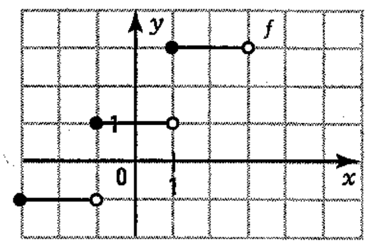
2. $x \in [-3, +\infty[$

d) $f(x) = 3\left[\frac{-1}{2}(x+2)\right] - 4$

1. $x \in]-\infty, -6]$

2. $x \in]-6, +\infty[$

7. Consider the function $f(x) = 2\left[\frac{1}{2}(x-1)\right] + 3$ represented on the right.



a) Represent the function $g(x) = 2\left[\frac{1}{2}(x+1)\right] + 1$.

What do you notice?

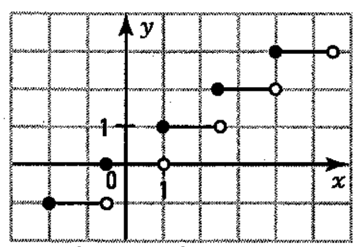
The graph of g coincides with the graph of f.

b) Find the rule of a function h with the same graph as f.

For example, $h(x) = 2\left[\frac{1}{2}(x+3)\right] - 1$

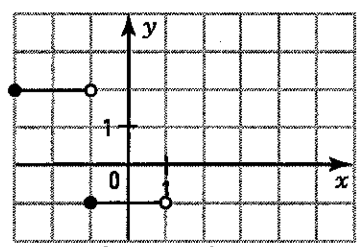
8. For each of the step functions represented below, find a rule corresponding to the function.

a)



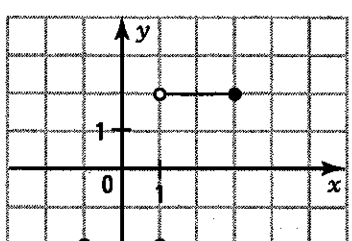
$y = \left[\frac{2}{3}(x-1)\right] + 1$

b)



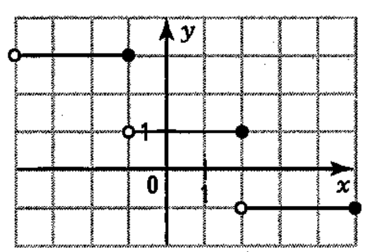
$y = -3\left[\frac{1}{2}(x+1)\right] - 1$

c)



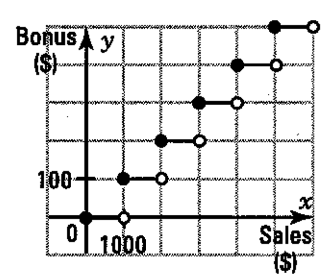
$y = -4\left[-\frac{1}{2}(x-1)\right] - 2$

d)



$y = 2\left[-\frac{1}{3}(x-2)\right] + 1$

9. To motivate his salesmen, a sales manager awards a bonus of \$100 for every \$1000 in sales.



a) Draw the Cartesian graph of the function which gives the awarded bonus y as a function of the amount of sales x.

b) What is the rule of the function? $y = 100\left[\frac{x}{1000}\right]$ $(x \geqslant 0)$

10. A salesman receives a weekly base salary of \$150 and a \$50 bonus for every \$1000 in weekly sales.

a) Find the rule of the function which gives the weekly salary y as a function of the amount x of weekly sales.

$y = 50\left[\frac{x}{1000}\right] + 150$

b) A salesman sells \$12 480 of merchandise in a week. What will his salary be?

\$750

c) For what amount of sales will the salesman receive a salary of \$1000?

An amount within the interval [17 000, 18 000[

d) Is it possible for a salesman to receive a salary equal to \$825?

$50\left[\frac{x}{1000}\right] + 150 = 825 \Leftrightarrow \left[\frac{x}{1000}\right] = 13.5$

The last equation has no solution since $13.5 \notin \mathbb{N}$.

11. The cost y, in dollars, of mailing a package depends on its mass x, in grams. This cost is defined by the rule $y = -2.5\left[-\frac{x}{100}\right]$.

a) What is the cost of mailing a package with a mass of 260 g? ***\$7.50***

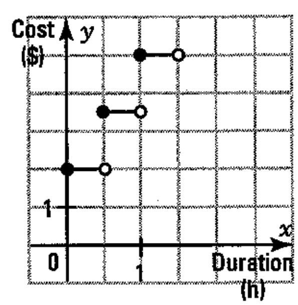
b) What is the mass of a package that costs \$12.50 to mail?

$x \in]400, 500]$

c) Explain in your words how to calculate the cost of mailing a package.

It costs \$2.50 for 100 g or less and \$2.50 more for every additional 100 g.

12. At a parking lot, the cost y of parking is calculated as follows: a minimum cost of \$2 for a parking time of less than 30 min. In addition, \$1.50 is charged for every 30 minute interval of parking time.



a) Draw the graph of the function which gives the total cost y, in dollars, of parking as a function of the parking time x, in hours.

b) Find the rule of the function.

$y = 1.5[2x] + 2$

c) What is the parking time corresponding to a cost of \$8?

$x \in [2, 2.5[$