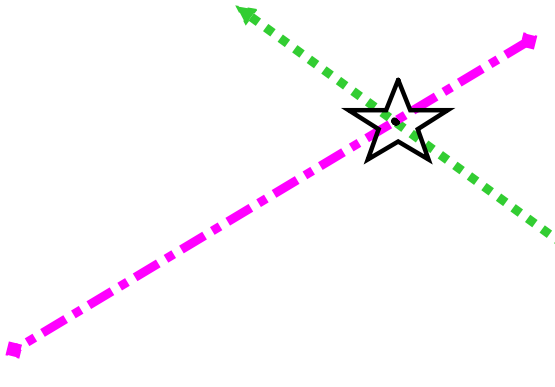
Solving Systems of Linear Relations

Finding where two lines intersect



"a.k.a."
corner
or
vertex

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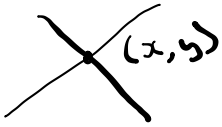
What is happening on the graph when two lines intersect?

In other words...

What do you notice about the x and y values for both lines? Same



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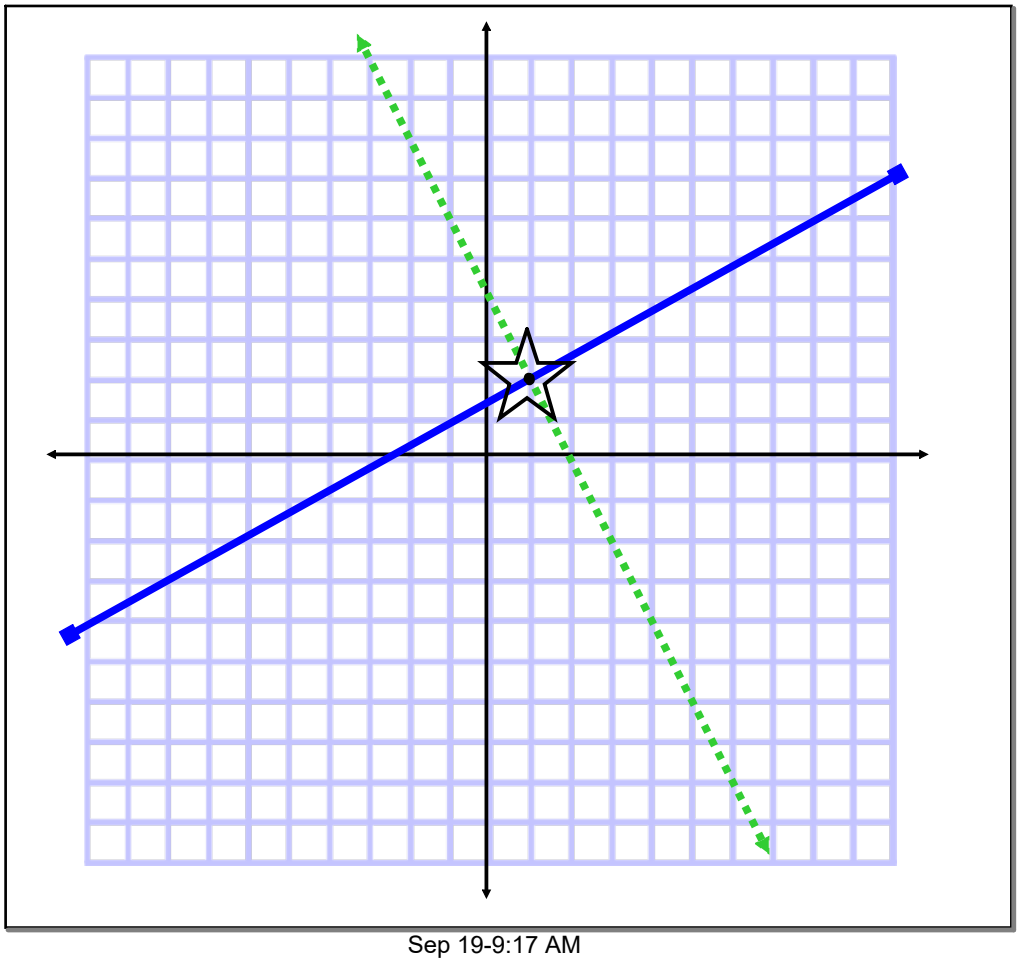

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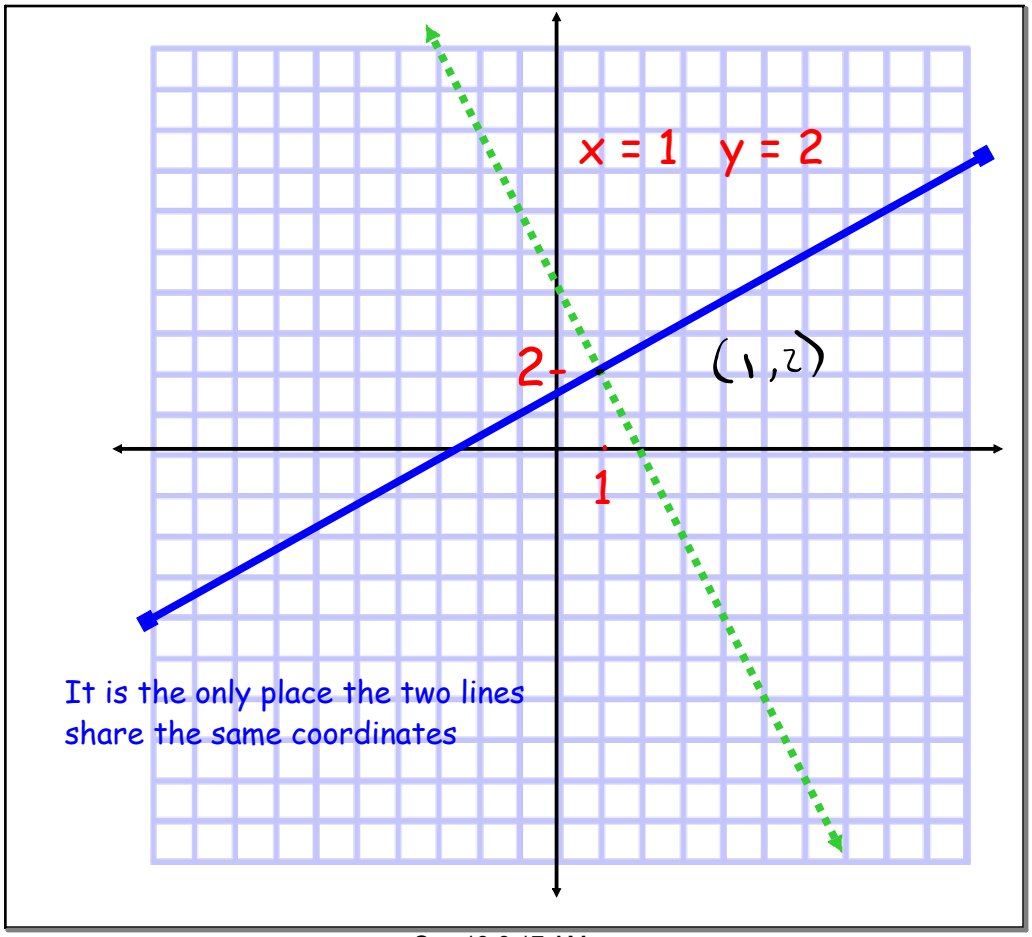

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Special cases

Coinciding lines are 2 lines that share the same equation

ex: $y = 2x + 6$

$\frac{2y}{2} = \frac{4x}{2} + \frac{12}{2} \Rightarrow y = 2x + 6$

These lines are ALWAYS intersecting

in fact they are the same line $\therefore \infty$ solutions

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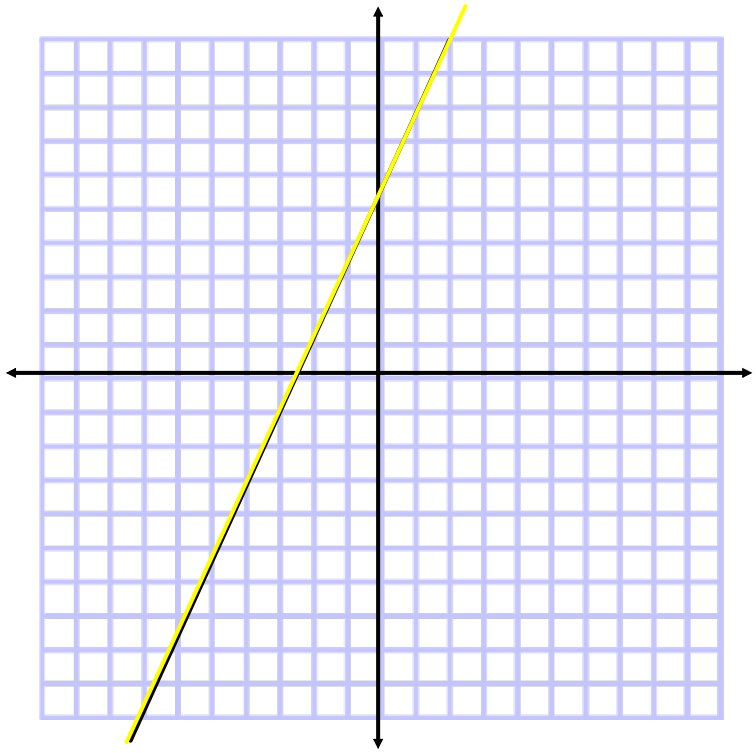

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Parallel lines never intersect

Parallel lines are.....

lines that have the same slope ∴ ∅ soln

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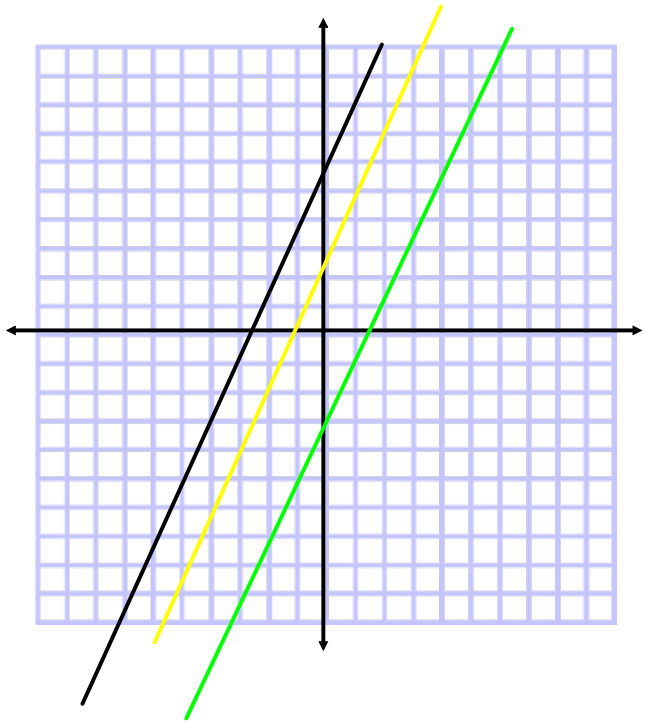

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1- Sometimes we can read the coordinates on the graph, like we just saw

2-Sometimes we need a table of values

What is the values of x and y where both lines are at the same point?

x	y_1	y_2
-2	-3	7
-1	-1	4
0	1	1
1	3	-2
2	5	-5

(0,1)

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3- Sometimes, we must solve it using algebra....

there are 3 ways of solving algebraically:

1 - Comparison method:

-when both equations are in function form

$y = ax + b$

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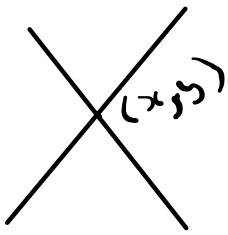
Ex. $y = 2x + 7$

$y = -x - 5$



Set the right side of the two equations equal to each other

We can do that because we are finding the value of x for both when the y value is the same for both.

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$2x + 7 = -x - 5$ solve for x

$2x + x = -7 - 5$ combine like terms

$3x = -12$

$x = (-12)/3 = -4$

Answer (-4, Y)

We still need to find y - so plug -4 into one of the equations (ie replace x with -4)

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$y = 2x + 7$

$y = 2(-4) + 7$

$y = -8 + 7$

$y = -1$

Verify with the other equation:

$y = -x - 5$

$y = -(-4) - 5$

$y = +4 - 5$

$y = -1$

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The final answer

$(-4, -1)$

Does this make sense?

Let's graph the lines and see

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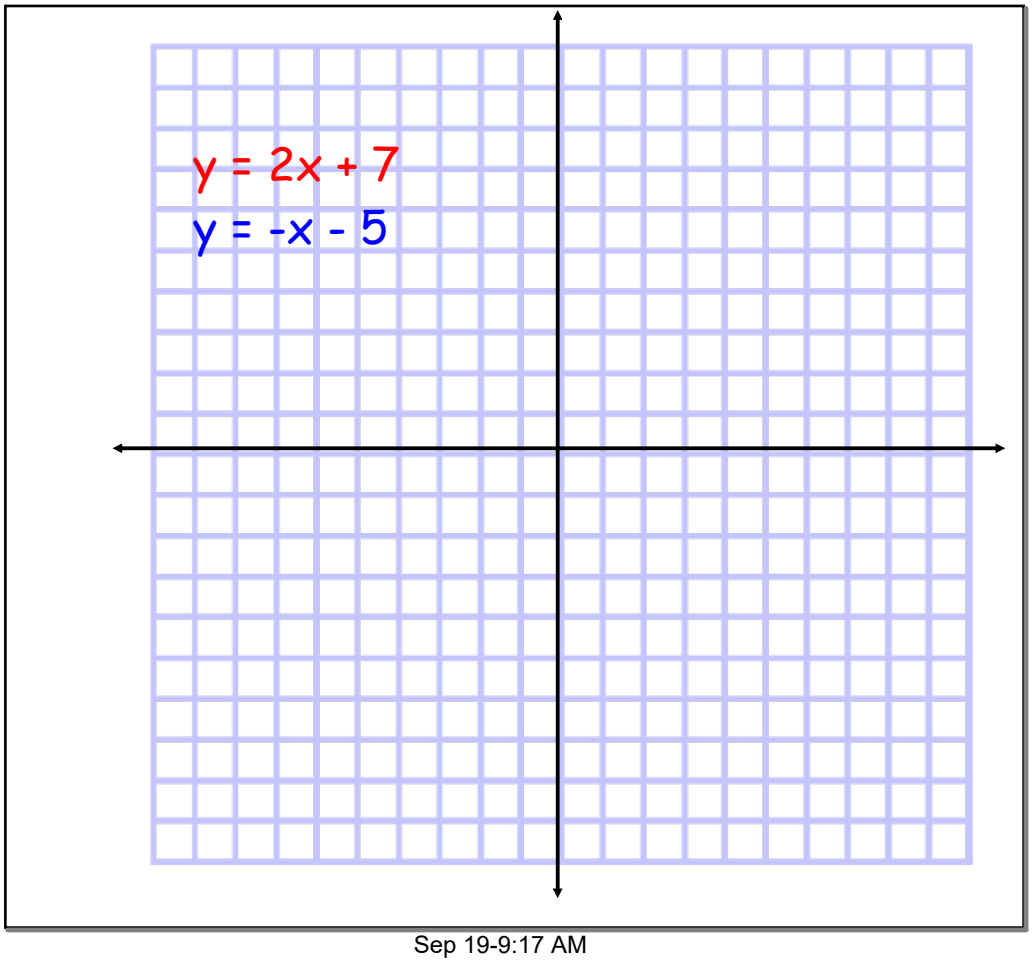

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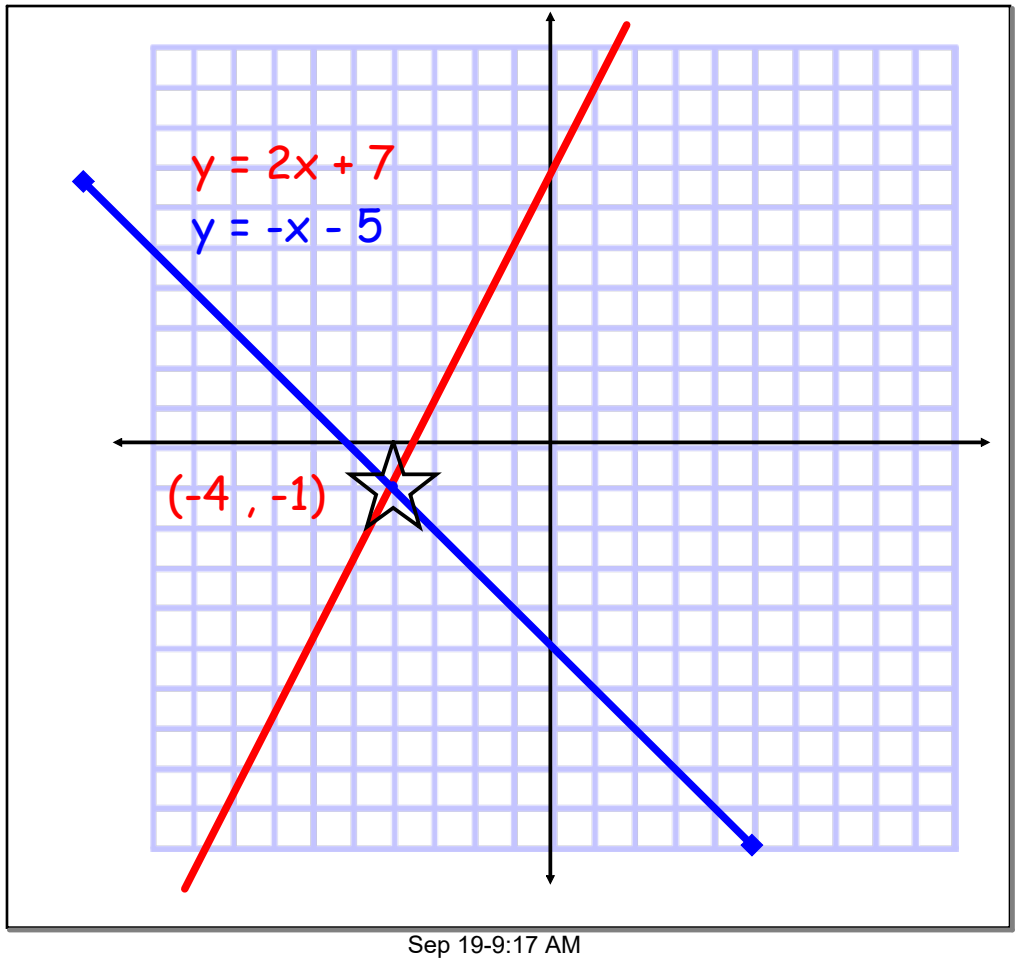

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Ex.1 Solve the following system:

$y = -x + 11$

$y = 0.5x + 2$

① $y = y$

$-x + 11 = 0.5x + 2$

$\frac{-1.5x}{-1.5} = \frac{-9}{-1.5}$

$x = 6$

② Plug it in: $y = -(6) + 11 = 5$

③ Check: $y = 0.5(6) + 2 = 5$ ✓ $(6, 5)$

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Ex.2 Solve the following system:

$y = 3x - 6$

$y = x - 2$

① $y = y$

$3x - 6 = x - 2$

$2x = 4$

$x = 2$

② Plug: $y = 3(2) - 6 = 0$

③ Check: $y = 2 - 2 = 0$ ✓

$(2, 0)$

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Ex.3 Solve the following system:

$y = 1.5x + 7$

$y = -x - 3$

① $y = y$

$1.5x + 7 = -x - 3$

$\frac{2.5x}{2.5} = \frac{-10}{2.5}$

$x = -4$

② Plug:

$y = 1.5(-4) + 7$

$= -6 + 7 = 1$

③ Check: $y = -(-4) - 3$

$= 4 - 3 = 1$ ✓

$(-4, 1)$

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Ex.4 Solve the following system:

$x = 2y + 4$

$x = y - 1$

(x, y)

① $x = x$

$2y + 4 = y - 1$

$y = -5$

② Plug:

$x = 2(-5) + 4$

$= -10 + 4 = -6$

③ Check: $x = -5 - 1 = -6$ ✓

$(-6, -5)$

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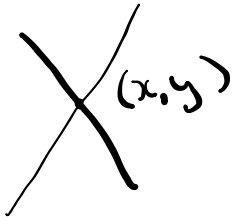
2 - Substitution method

Sometimes one equation is in function form and one is not....

$Ax + By = C$

$y = ax + b$

$Ax + B(ax + b) = C$



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$9x + 3y = 18$

$y = 2x + 6$

lonely y

Replace the y in the first equation with what the lonely y is equal to

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$9x + 3(2x + 6) = 18$

multiply 3 into the brackets...

$9x + 6x + 18 = 18$

combine like terms

$15x = 18 - 18$

$15x = 0$

Divide by 15

$x = 0/15 = 0$

$x = 0$

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$9x + 3y = 18$

$y = 2x + 6$

$x = 0$

What is Y?

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$9(0) + 3y = 18$

$0 + 3y = 18$

$3y = 18$

$y = 6$

Now what?

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Verify with the second equation...

$y = 2x + 6$

$= 2(0) + 6 = 6$

(0,6)

$6 = 2(0) + 6$

$6 = 6$ True

Final answer: (0,6)

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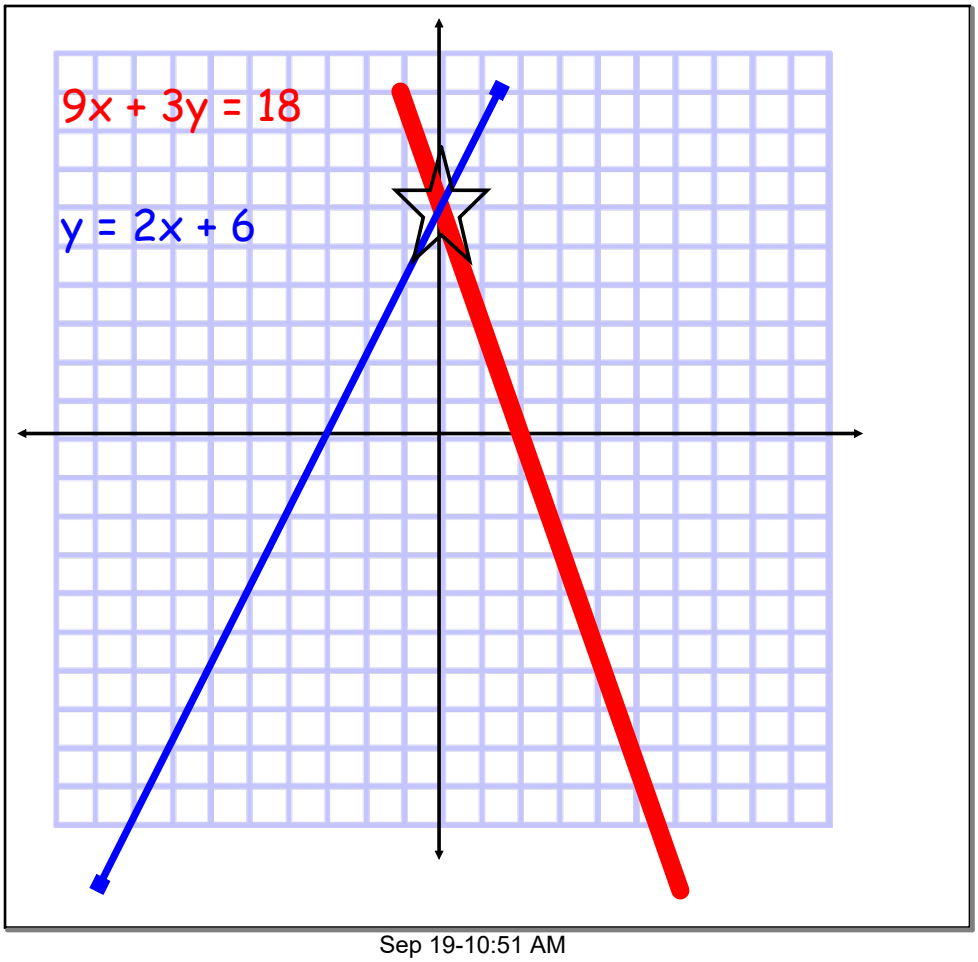

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3 - Elimination Method

If the equations are both in general form then use elimination method (also called addition)

$Ax + By + C = 0$

or perhaps it might look like...

$Ax + By = C$

(ie. x and y are on the same side of the equation)

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Be sure to line up all the like terms so they are above/below each other

Ex.

$$\begin{array}{r} 6x + 2y = 10 \\ -2x + 2y = 2 \\ \hline 8x \qquad\quad = 8 \end{array}$$

$x = 1$

Look at the values in front of the y

They have the same coefficient... (2)
We want to eliminate y when we add the 2 equations so we want them to have opposite signs

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$6x + 2y = 10$
$-2x + 2y = 2$

What can we do?

multiply one of the equations by -1 which will simply change all the signs

Pull

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$6x + 2y = 10$
$2x + -2y = -2$

Now, add them up

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$x = 1$

Find Y

$6x + 2y = 10$
$-2x + 2y = 2$

Use the first equation

$6(1) + 2y = 10$
$6 + 2y = 10$
$2y = 4$
$y = 2$

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x = 1 y = 2

Answer (1,2)

Verify with the second equation

$6x + 2y = 10$

$-2x + 2y = 2$

$-2(1) + 2(2) = 2$

$-2 + 4 = 2$

$2 = 2$ true

the final answer: (1,2) ;)

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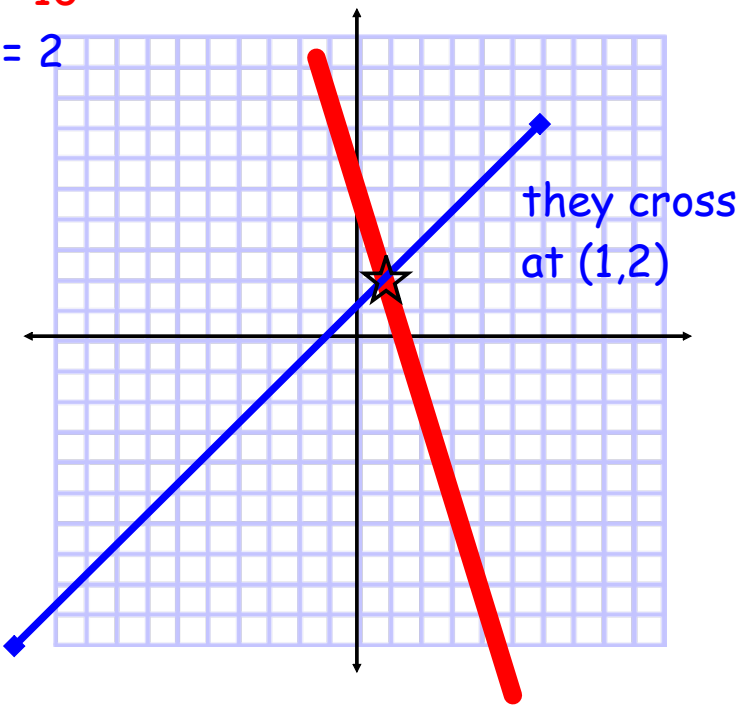
Does this make sense on the graph?

$6x + 2y = 10$

$-2x + 2y = 2$



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If the coefficients are different we need to multiply one or both equations to make the cofficients of ONE of the variables the same.

Ex. Solve the following system:

$5(3x - 2y = 2) \Rightarrow 15x - 10y = 10$

$3(5x - 5y = 10) \Rightarrow -(15x - 15y = 30)$

$5y = -20$

$y = -4$

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Homework

P. 169 #1

P. 171 #3

P. 172 #4 & 5

P. 176 #6

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