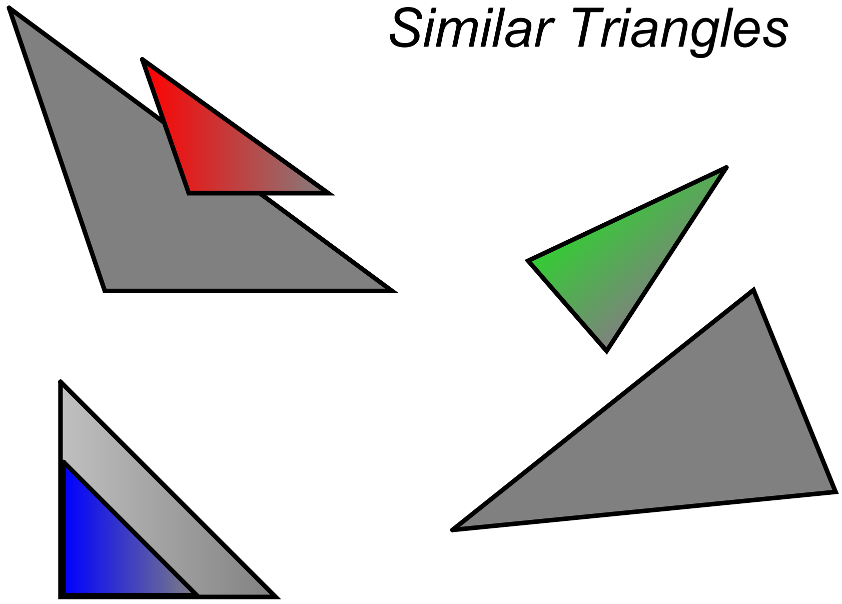
Triangle Geometry

Similar Triangles



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similarity has a lot to do with proportionality

the mathematical understanding of SIMILAR is different than our "normal" understanding of the word

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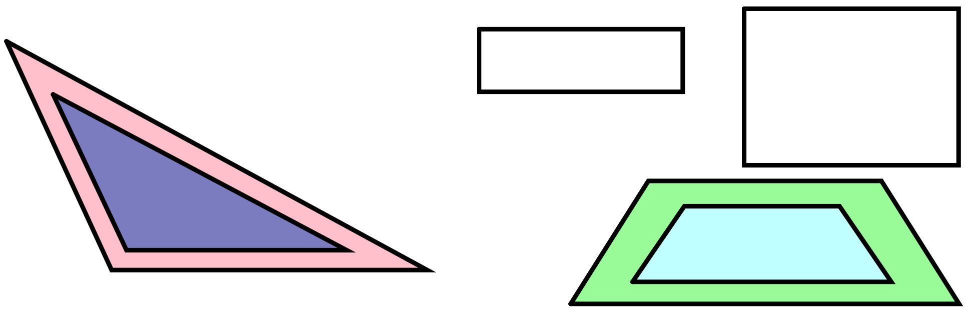
Two figures are considered SIMILAR if:

....they have the same **shape**
....the corresponding angles are **congruent**
....the corresponding sides are **proportional**



All three of the above conditions must be met

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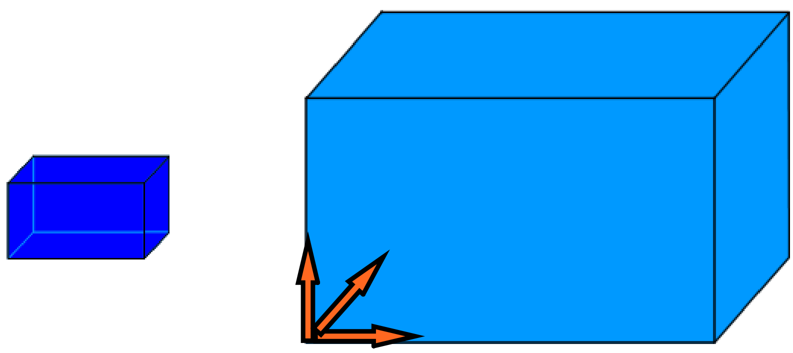
When things grow **proportionally**, we say that they grow in all dimensions by the *same factor*

if something gets three times as wide, it will also get three times as tall and three times as deep



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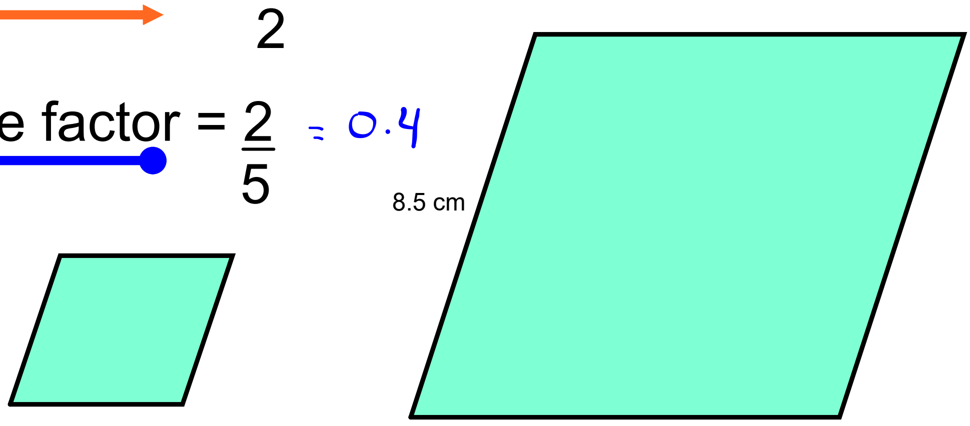
If two parallelograms are proportional, then the corresponding sides are larger (or smaller) by the same **factor**.

$$\frac{3.4}{8.5} \overset{?}{=} \frac{3}{7.5}$$

scale factor = $\frac{5}{2}$ = 2.5

scale factor = $\frac{2}{5}$ = 0.4



these are proportional

increasing the size, then k > 1 decreasing the size, then k < 1

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A proportion is made of two equal ratios (fractions)

$$\frac{1}{2}=\frac{2}{4}$$

from the previous page: $\frac{3.4}{8.5}=\frac{3}{7.5}=\frac{\text{the measure of a side from "the first" figure}}{\text{the measure of its corresponding side in the second figure}}$

Mar 23-10:19 AM

$$\boxed{\frac{a}{b} = \frac{c}{d}}$$

a proportion

after we cross multiply, these will be equal

$$\boxed{ad=bc}$$

ex

$$\frac{3.4}{8.5} = \frac{3}{7.5}$$

$3.4(7.5) = 8.5(3)$

$25.5 = 25.5$

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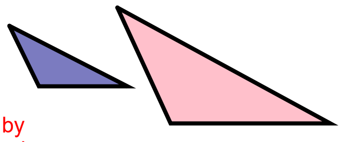
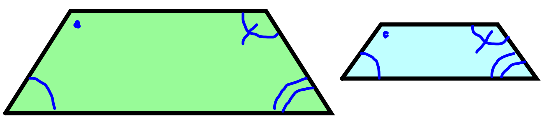
Two figures are considered **SIMILAR** if

....they have the same **shape**



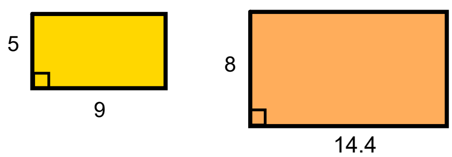
this you can verify by looking...is it a triangle, trapezoid, square etc...

....the corresponding angles are **congruent**



this you can verify mathematically ...the measures are either given or can be calculated or deduced

....the corresponding sides are **proportional**



this you can verify mathematically ...set up a proportion

$$\frac{5}{8} \stackrel{?}{=} \frac{9}{14.4}$$

$72 = 72$ ✓

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Remember!

Once a triangle has been proven to be similar to another triangle, then you know two things:

- corresponding angles are **congruent**
- corresponding sides are **proportional**

Mar 23-6:28 PM

Like proving congruence...we also have minimum conditions to *prove* **SIMILARITY**

Mar 23-10:50 AM

3 ways to prove that triangles are similar: $\sim$

1. AA -- angle-angle
2. SSS_{prop} -- side-side-side (prop)
3. SAS_{prop} -- side-angle-side (prop)

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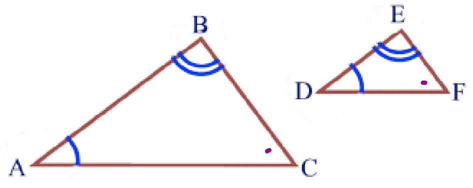
1. Proving triangles are similar by AA

Angle-Angle



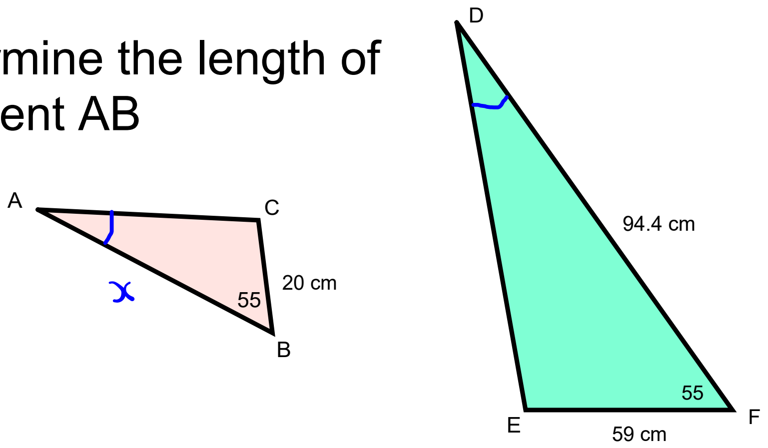
		statement	Justification
A	1	$\angle A \cong \angle D$	Hyp. (given)
A	2	$\angle B \cong \angle E$	" "
	3	$\triangle ABC \sim \triangle DEF$	AA

$$\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AC}}{\overline{DF}}$$

Mar 23-12:45 PM

Example of a question:

Determine the length of segment AB



Step 1: Redraw the triangles so that both have the same orientation

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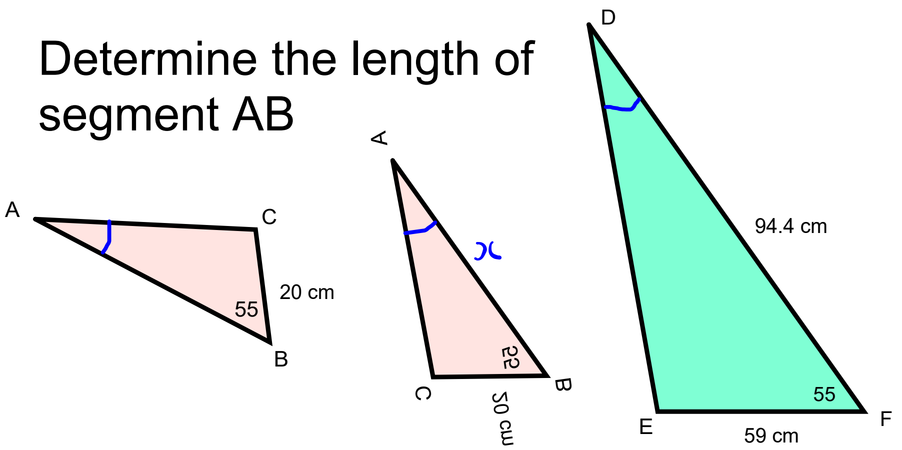
Example of a question:

Determine the length of segment AB



		statement	justification
A	1	∠A ≅ ∠D	Hyp. (given)
A	2	∠B ≅ ∠F	" "
	3	△ABC ~ △DEF	AA
	4	$\overline{AB}$ = 32 cm	Corr to $\overline{DF}$ in Similar △

$$\frac{20}{59} = \frac{x}{94.4}$$

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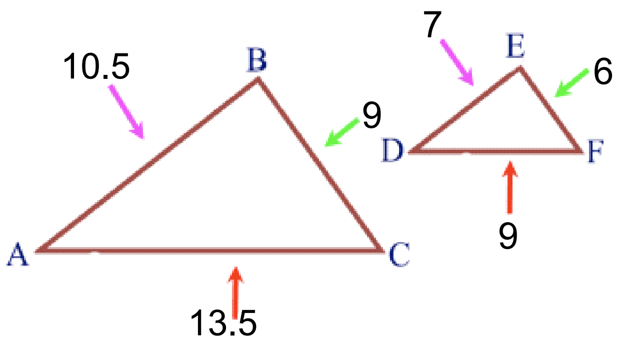
2. Proving triangles are similar by **SSSprop**

side-side-side (prop)



	Statement	Justification
1	$\frac{DE}{AB} \cong \frac{EF}{BC}$ $\left(\frac{7}{10.5} = \frac{6}{9}\right)$	Corr sides are prop.
2	$\frac{DE}{AB} \cong \frac{DF}{AC}$ $\left(\frac{7}{10.5} = \frac{9}{13.5}\right)$	" " " "
3	$\frac{EF}{BC} \cong \frac{DF}{AC}$ $\left(\frac{6}{9} = \frac{9}{13.5}\right)$	" " " "
4	$\triangle ABC \sim \triangle DEF$	SSS prop

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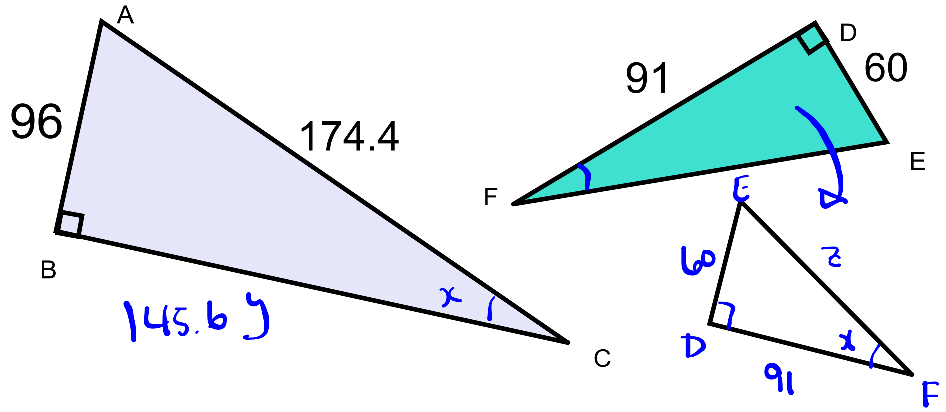
Example of a question: Prove that C F



	statement	justification
1	$m\overline{BC} = 145.6$	$\triangle ABC$ is right $\triangle$ (Pyth)
2	$\frac{\overline{ED}}{\overline{AB}} = \frac{\overline{DF}}{\overline{BC}}$	$\frac{60}{96} = \frac{91}{145.6} = \frac{5}{8}$
3	$\angle ABC \cong \angle EDF$	Hyp (Given)
4	$\triangle ABC \sim \triangle DEF$	SAS prop
5	$\angle C \cong \angle F$	Corr. $\angle$ in Sim $\triangle$s

Mar 23-12:51 PM

Homework

Workbook

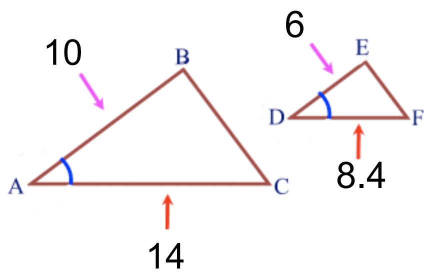
P204 #2

P 206 #7 + 8

Mar 23-2:02 PM

3. Proving triangles are similar by **SASprop**

side-angle-side (prop)



	statement	justification

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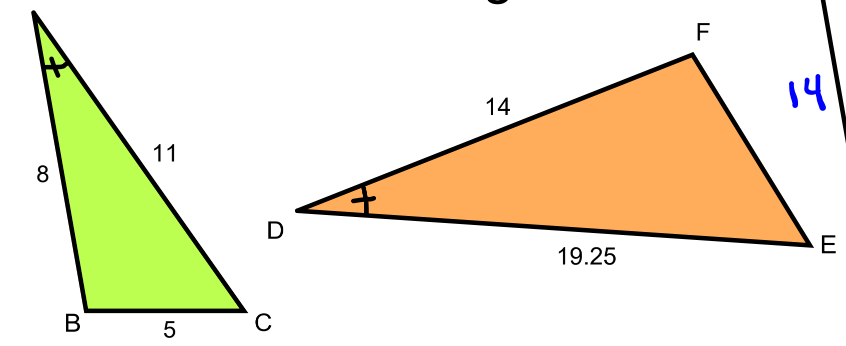
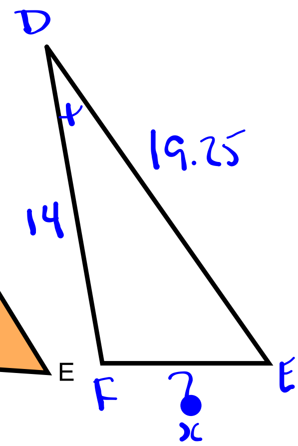
Example of a question:

What is the length of $\overline{EF}$?





	statement	justification
1	$\frac{\overline{AB}}{\overline{DF}} = \frac{\overline{AC}}{\overline{DE}}$	$\frac{8}{14} = \frac{11}{19.25} = \frac{4}{7}$ Corr sides are prop.
2	$\angle A \cong \angle D$	Hyp (given)
3	$\Delta ABC \sim \Delta DEF$	SAS prop
4	$m\overline{EF} = 8.75$	$\frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AB}}{\overline{DF}} \rightarrow \frac{5}{x} = \frac{8}{14}$

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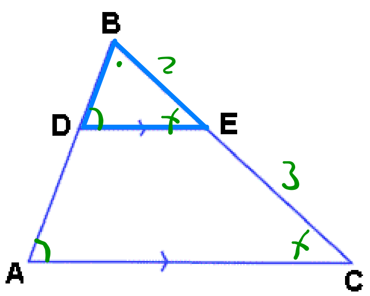
Dealing with OVERLAPPING TRIANGLES



Many problems involving similar triangles have one triangle ON TOP OF (overlapping) another triangle.

Here we have ΔBDE and ΔBAC

In this case line segments DE and AC are shown to be parallel--note the arrows on the line segments.

so,
we know that <*BDE* is congruent to <*DAC* (by corresponding angles).

<*B* is shared by both triangles

so the two triangles are similar by AA

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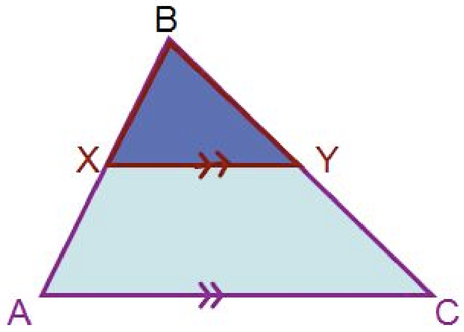
Parallel Line to a Triangle's Side: Any line parallel to a triangle's side determines two similar triangles.



$\Delta BXY \quad \Delta BAC$

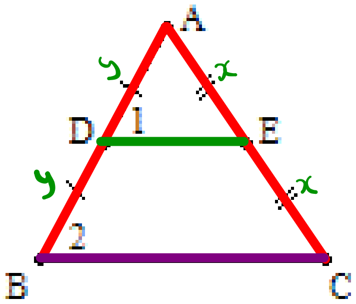
angles are congruent
sides are proportional

Corollary: Any two similar triangle determine parallel lines.

Mar 22-4:16 PM

Segment Joining the Midpoints of Two Sides in a Triangle: Any segment joining the midpoints of two sides in a triangle is parallel to the third side and is half the measure of this third side.

$$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}}$$



In triangle ABC

If: $AD = DB$, $AE = EC$

Then: $DE \parallel BC$

$$\overline{DE} = \frac{1}{2}\overline{BC}$$

$$\frac{y}{2y} = \frac{x}{2x} = \frac{1}{2}$$

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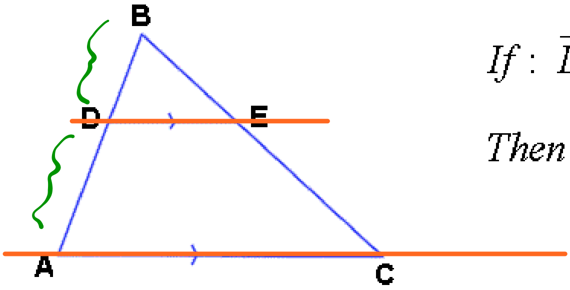
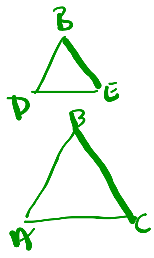
Thales' Theorem: Two intersecting transversal lines intersected by parallel lines are separated into corresponding segments of proportional length.



$If: \overline{DE} \parallel \overline{AC}$

$Then: \frac{BD}{DA} = \frac{BE}{EC} \neq \frac{\overline{DE}}{\overline{AC}}$



$\frac{\overline{DE}}{\overline{AC}} = \frac{\overline{BD}}{\overline{BA}} = \frac{\overline{BE}}{\overline{BC}}$

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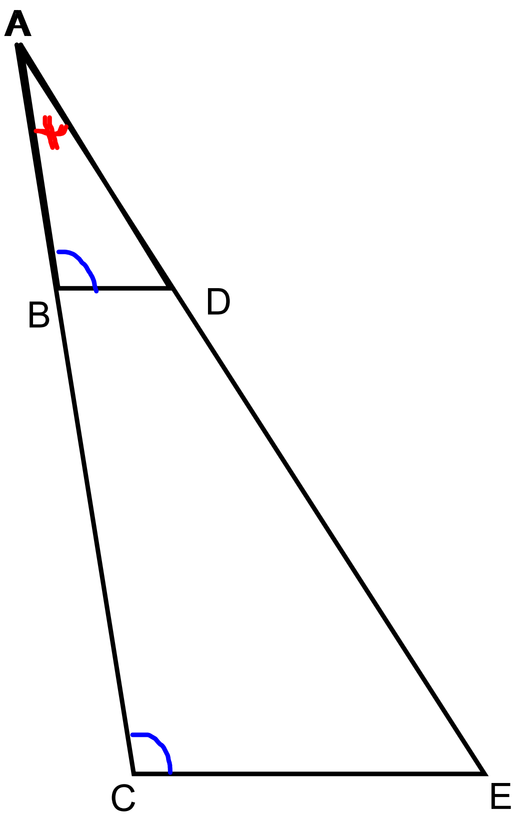

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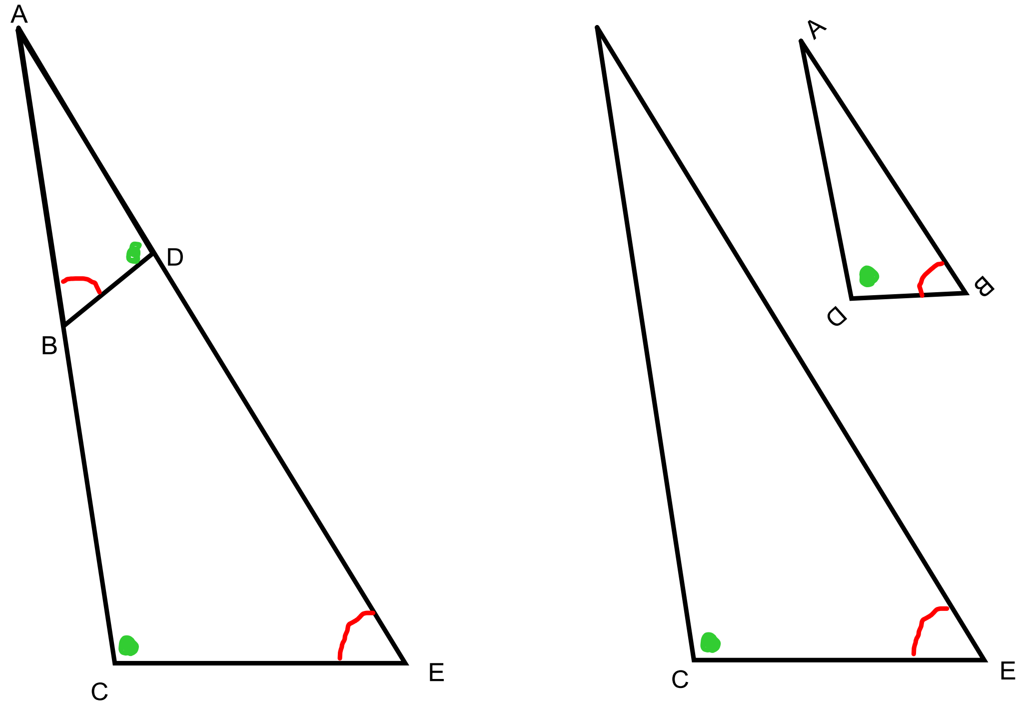
More OVERLAPPING TRIANGLES



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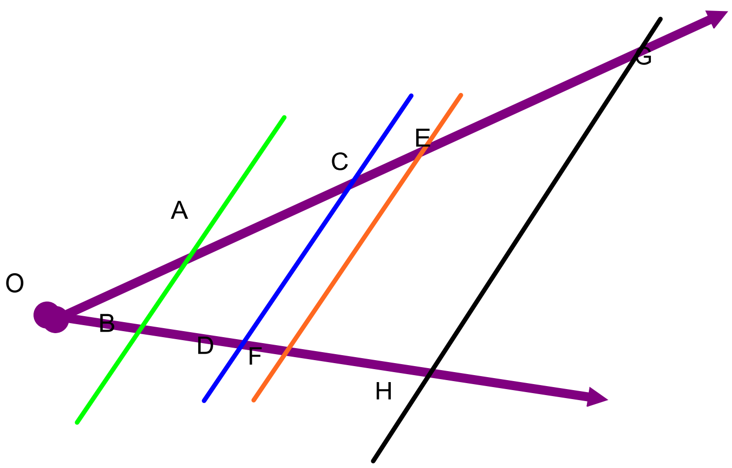
Lines AB CD EF GH

Does not have to be a triangle



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Homework

Workbook:

P. 207 #9

P. 209 #12

P. 212 #2

P. 214 #5-10

Mar 22-4:17 PM