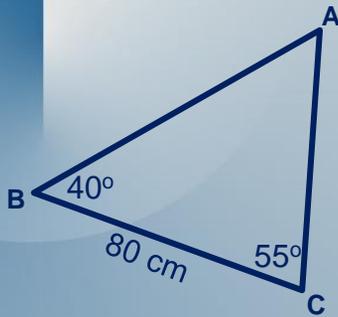


Warm Up

Find $m\overline{AB}$.



$$\angle BAC = 180 - 40 - 55 = 85^\circ$$

$$\Rightarrow \frac{\overline{AB}}{\sin 55} = \frac{80}{\sin 85}$$

$$\therefore m\overline{AB} = \frac{80 \sin 55}{\sin 85} = 65.78\text{m}$$

Lesson 36

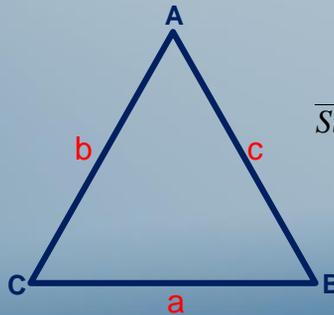
Trigonometry – Cosine Law

Trigonometry

We use sine law when we know:

- an **angle** and the **side opposite** to it, and
- one other piece of information

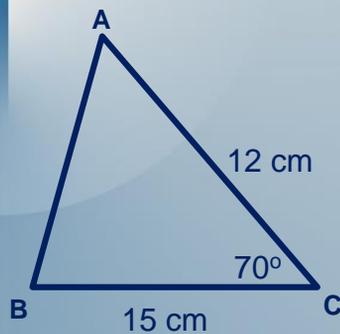
We have one complete ratio and half of a second ratio.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

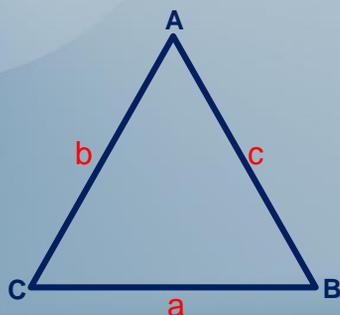
Example 1

Find $m\overline{AB}$.



Cosine Law

The square of any side in a triangle is equal to the sum of the squares of the other two sides minus twice the product of these sides by the cosine of the angle between these two sides.



$$a^2 = b^2 + c^2 - (2bc \cos A)$$

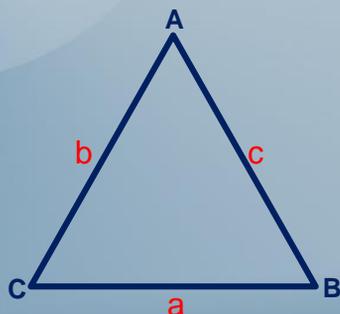
$$b^2 = a^2 + c^2 - (2ac \cos B)$$

$$c^2 = a^2 + b^2 - (2ab \cos C)$$

Cosine Law

Case 1: Finding a SIDE given 2 sides and the angle between the given sides.

Case 2: Finding an ANGLE given 3 sides.



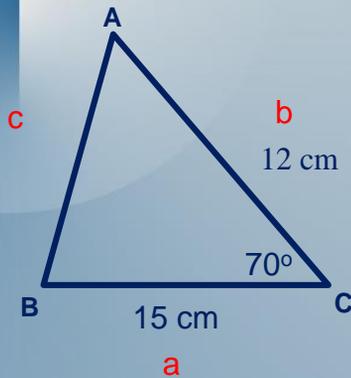
$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$b^2 = a^2 + c^2 - (2ac \cos B)$$

$$c^2 = a^2 + b^2 - (2ab \cos C)$$

Example 1

Find $m\overline{AB}$.



$$c^2 = a^2 + b^2 - (2ab \cos C)$$

$$c^2 = 15^2 + 12^2 - (2(15)(12) \cos 70^\circ)$$

$$c^2 = 225 + 144 - (360 \cos 70^\circ)$$

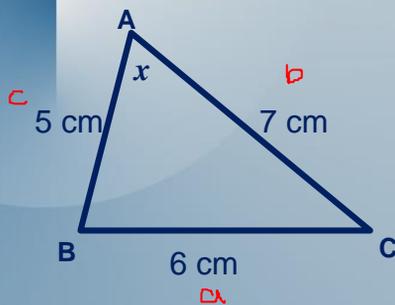
$$c^2 = 225 + 144 - 123.13$$

$$\Rightarrow c = \sqrt{245.87}$$

$$\therefore c \approx 15.68 \text{ cm}$$

Example 2

Find $\angle BAC$.



$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$6^2 = 7^2 + 5^2 - (2(7)(5) \cos A)$$

$$36 = 49 + 25 - (70 \cos A)$$

$$-38 = -70 \cos A$$

$$\frac{-38}{-70} = \frac{-70 \cos A}{-70}$$

$$0.542857 = \cos A$$

$$\Rightarrow A = \cos^{-1}(0.542857)$$

$$\therefore m\angle BAC \approx 57^\circ$$

Cosine Law

Case 2: Finding an ANGLE given 3 sides.

We can derive:

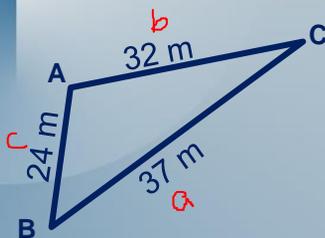
$$a^2 = b^2 + c^2 - (2bc \cos A) \Rightarrow \angle A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

$$b^2 = a^2 + c^2 - (2ac \cos B) \Rightarrow \angle B = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)$$

$$c^2 = a^2 + b^2 - (2ab \cos C) \Rightarrow \angle C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$$

Example 3

Find $\angle ABC$.



$$b^2 = a^2 + c^2 - (2ac \cos B)$$

$$32^2 = 37^2 + 24^2 - (2(37)(24) \cos B)$$

$$1024 = 1369 + 576 - (1776 \cos B)$$

$$-921 = -1776 \cos B$$

$$\frac{-921}{-1776} = \frac{-1776 \cos B}{-1776}$$

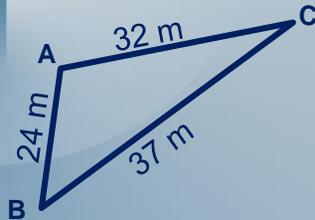
$$0.518581 = \cos B$$

$$B = \cos^{-1}(0.518581)$$

$$m\angle BAC \approx 58.76^\circ$$

Example 3

Find $\angle ABC$.



$$\angle B = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right)$$

$$\angle B = \cos^{-1}\left(\frac{32^2 - 37^2 - 24^2}{-2(37)(24)}\right)$$

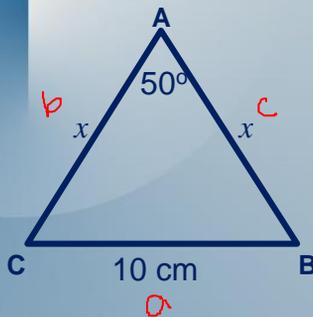
$$\angle B = \cos^{-1}\left(\frac{-921}{-1776}\right)$$

$$\angle B = \cos^{-1}(0.518581)$$

$$m\angle BAC \approx 58.76^\circ$$

Example 4

Use the cosine law to calculate the value of x ?



$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$10^2 = x^2 + x^2 - (2(x)(x) \cos 50)$$

$$100 = 2x^2 - (2x^2)(0.642788)$$

$$100 = 2x^2 - (1.285576x^2)$$

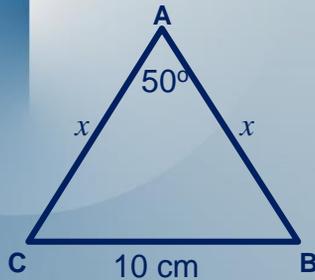
$$100 = 0.714424x^2$$

$$139.97 = x^2$$

$$\therefore x \approx 11.83 \text{ cm}$$

Example 5

Use the sine law to calculate the value of x ?



$$\angle BCA = \angle ABC = \frac{(180 - 50)}{2} = 65^\circ$$

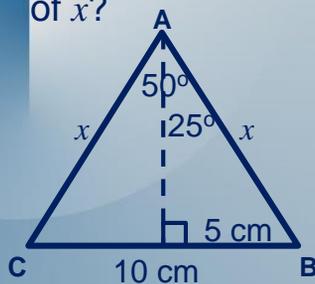
$$\Rightarrow \frac{x}{\sin 65} = \frac{10}{\sin 50}$$

$$x = \frac{10 \sin 65}{\sin 50}$$

$$\therefore x \approx 11.83 \text{ cm}$$

Example 6

Use trig ratios (SohCahToa) to calculate the value of x ?



$$\sin 25 = \frac{5}{x} \quad \text{or} \quad \cos 65 = \frac{5}{x}$$

$$\therefore x \approx 11.83 \text{ cm}$$

NB: We can use this method ONLY with isosceles triangles

Homework

Workbook

P. 243 #10 & 11

P. 244 #12, 14, 16 & 18