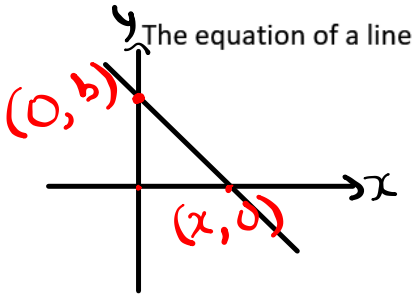
Equation of a Line

Recall : How to calculate the slope of a line given two points

(x_1, y_1) and (x_2, y_2) Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$



The equation of a line in **functional form**:

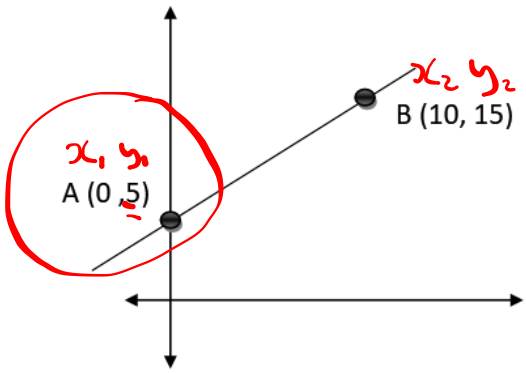
$y = ax + b$

b= y-intercept or initial value

a = slope

To determine the equation of a line from a graph:

Example # 1



$y = ax + b$

(1) $a = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{15 - 5}{10 - 0} \rightarrow \frac{10}{10} = 1$

(2) $y = 1x + b$

$y = x + 5$

Step 1 – Pick and label two points as (x_1, y_1) and (x_2, y_2)

(0,5) is (x_1, y_1) & (10,15) is (x_2,y_2)

Step 2 – Determine the slope of the line

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1} = \frac{15-5}{10-0} = \frac{10}{10} = 1$$

Step 3 – Plug in a point into the BASE equation to replace the x and the y with actual numbers (also plug in the slope that you just found for the "a") to solve.

$$Y = ax + b$$

$$15 = 1(10) + b$$

$$15 = 10 + b$$

$$15 - 10 = b$$

$$5 = b$$

Step 4 – Re-write the equation with the "a" and "b" filled in (leave the x and y)

$$Y = 1x + 5$$

$\frac{-6}{8} = \frac{6}{-8} = -\frac{6}{8}$

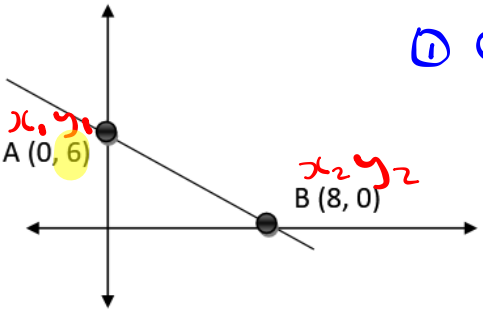
$y = ax + b$

① $a = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-6}{8-0} = \frac{-6}{8}$

$= -\frac{3}{4} = -0.75$

Example # 2



$b = 6$

$y = -\frac{3}{4}x + 6$

Step 1 – Pick and label two points as (x_1, y_1) and (x_2, y_2)

(0,6) is (x_1, y_1) & (8,0) is (x_2,y_2)

Step 2 – Determine the slope of the line

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1} = \frac{0-6}{8-0} = \frac{-6}{8} = \frac{-3}{4}$$

Step 3 – Plug in a point into the BASE equation to replace the x and the y with actual numbers (also plug in the slope that you just found for the "a") AND SOLVE

$$Y = ax + b$$

$$0 = \frac{-3}{4}(8) + b$$

$$0 = -6 + b$$

$$0 + 6 = b$$

$$6 = b$$

Step 5 – Re-write the equation with the "a" and "b" filled in (leave the x and y)

$$y = \frac{-3}{4}x + 6$$

$a = \frac{y_2 - y_1}{x_2 - x_1}$

Example #3

$\frac{2}{3} \neq 0.67$

$\neq 0.\overline{66}$

x	y
9	2
3	-2
-3	-6
-9	-10

x_1 y_1

x_2 y_2

$= \frac{-2-2}{3-9} = \frac{-4}{-6} = \frac{2}{3}$

Step 1 – Pick and label two points as (x_1, y_1) and (x_2, y_2)

(9,2) is (x_1, y_1) & (-9,-10) is (x_2,y_2)

Step 2 – Determine the slope of the line

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1} = \frac{-10-2}{-9-9} = \frac{-12}{-18} = \frac{2}{3}$$

Step 3 – Plug in a point into the BASE equation to replace the x and the y with actual numbers (also plug in the slope that you just found for the "a")

$$Y = ax + b$$

$$-6 = \frac{2}{3}(-3) + b$$

Step 4 – Simplify the equation and isolate the "b" to solve for the variable

$$-6 = \frac{2}{3}(-3) + b$$

$$-6 = -2 + b$$

$$-6 + 2 = b$$

$$-4 = b$$

Step 5 – Re-write the equation with the "a" and "b" filled in (leave the x and y)

$$y = \frac{2}{3}x - 4$$